Nonparametric Dynamic Conditional Beta

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Contribution

- Jointly model firm and market excess returns
  - Semiparametric multivariate GARCH model (MGARCH).
  - Use a Dirichlet process mixture (DPM) model for state-specific parameters for mean and covariance matrix.
- From this model derive the conditional distribution of firm returns
  - The conditional distribution completely determines pricing.
  - This is a nonparametric model that depends on market excess returns
  - Beta is time-varying and linked to the conditional second moment.
  - Beta depends nonlinearly on the contemporaneous market excess return and other factors.
- Semiparametric MGARCH model dominates benchmarks models.
- Find significant nonlinear dependence in beta as a function of factors.

Estimating the Beta coefficient

$$r_{it}^e = \alpha_i + \beta_i r_{mt}^e + \epsilon_{it}$$

- Regressing the excess stock returns on the excess market return as in the one-factor CAPM (Sharpe 1964, Lintner 1965).
- Fama-French three-factor model (Fama and French 1993).

Beta is not constant

- The role of conditioning information in asset pricing models: Hansen and Richard (1987), Jagannathan and Wang (1996),...
- Time-varying beta: Blume (1975), Fabozzi and Francis (1978), Bos and Newbold (1984), Bollerslev et al. (1988), Lettau and Ludvigson (2001), and Ang and Chen (2007)

Estimating dynamic conditional beta

- Rolling linear regression, Fama and Macbeth (1973), Fama and French (1997)
- State-space framework: Bos and Newbold (1984), Collins et al. (1987), Faf et al. (1992), Gao (2004), Adrian and Franzoni (2009)

$$
\begin{aligned}
r_t^e &= \beta_t r_{mt}^e + \eta_t \\
\beta_t &= F\beta_{t-1} + \epsilon_t
\end{aligned}
$$

- Markov switching models: Fridman (1994), Fabozzi and Francis (1977), Huang (2000, 2001), Huang and Hueng (2008), Abdymomunov and Morley (2011)
- MGARCH, link conditional second moments to dynamic beta: Giannopoulos (1995), Bollerslev et al. (1988), McCurdy and Morgan (1991, 1992), Hansen et al. (2014)

Benchmark model

- y_t excess stock return on an asset
- $x_t = (x_{1,t}, ..., x_{q,t})'$ q regressors (factors) including the excess market return
- $r_t = (y_t, x_t')'$

MGARCH-t: extend Engle (2016) to Student-t.

$$r_t | r_{1:t-1} \sim t(\mu, H_t, \nu),$$
$$H_t = \Gamma_0 + \Gamma_1 \odot (r_{t-1} - \eta)(r_{t-1} - \eta)' + \Gamma_2 \odot H_{t-1},$$

$\Gamma_0 = \Gamma_0^{1/2}(\Gamma_0^{1/2})'$, $\Gamma_1 = \gamma_1(\gamma_1)'$, and $\Gamma_2 = \gamma_2(\gamma_2)'$, γ_1, γ_2 are $q+1$ vectors.

Benchmark model

The conditional density of y_t given x_t:

$$
\begin{aligned}
y_t|x_t &\sim t(\mu_{y|x}, H_{t,y|x}, \nu_{y|x}), \\
\mu_{y|x} &= \mu_y + H_{yx,t}H_{xx,t}^{-1}(x_t - \mu_x), \\
H_{t,y|x} &= \frac{\nu + (x_t - \mu_x)' H_{xx,t}^{-1}(x_t - \mu_x)}{\nu + q}(H_{yy,t} - H_{yx,t}H_{xx,t}^{-1}H_{yx,t}'), \\
\nu_{y|x} &= \nu + q,
\end{aligned}
$$

Conditioning on one factor, the excess market return, $x_t \equiv x_{m,t}$,

$$E[y_t|x_{m,t}, H_t] = \mu_y + H_{ym,t}H_{mm,t}^{-1}(x_{m,t} - \mu_m).$$

$$\beta_t \equiv \frac{\partial E[y_t|x_{m,t}, H_t]}{\partial x_{m,t}} = H_{ym,t}H_{mm,t}^{-1}.$$

Conditional on $r_{1:t-1}$ we have the usual conditional CAPM

$$E[y_t|r_{1:t-1}] = \mu_y + \beta_t(E[x_{m,t}|r_{1:t-1}] - \mu_m).$$

β_t and Asset Pricing

β_t is consistent with a pricing kernel linear in the factors. Let the pricing kernel be defined as

$$m_t = a_t - b_t^{'} x_t.$$

Then in equilibrium an asset's excess return y_t obeys

$$E_{t-1}[y_t] = (1 + r_t^f) b_t \text{Cov}_{t-1}(x_t, r_t).$$

Assuming the factors are also priced by this kernel results in

$$\begin{aligned} E_{t-1}[y_t] &= \beta_t E_{t-1}[x_t - r_t^f], \\ \beta_t &= \frac{\text{Cov}_{t-1}(x_t, y_t)}{\text{Var}_{t-1}(x_t)}. \end{aligned}$$

Some Observations

- The joint distribution uniquely determines the conditional distribution and compensated risk.
- This decomposition applies more generally to elliptic family of distributions.
  - results in a conditionally linear pricing relationship in factors.
- We nonparametrically model the conditional distribution
  - Allow for deviations from elliptic distribution
  - Nest normal and Student-t cases
  - Conditional distribution will nonlinearly depend in conditioning variable $\rightarrow$ nonlinear factor model.

Nonparametric model: intuition

- Model $r_t = (y_t, x_{1,t}, ..., x_{q,t})'$ as infinite mixture

$$r_t | H_t, \mu, B, W \sim \sum_{j=1}^{\infty} \omega_j N(\mu_j, (H_t^{1/2}) B_j (H_t^{1/2})').$$

$H_t = H_t^{1/2}(H_t^{1/2})'$, $\sum_{j=1}^{\infty} \omega_j = 1$.

- Conditional distribution

$$p(y_t|x_t) = \sum_{j=1}^{\infty} q_j(x_t) f_{y_t|x_t}(y_t|x_t, \mu_j, B_j, H_t),$$

$$q_j(x_t) \propto \omega_j f_{x_t}(x_t|\mu_j, B_j, H_t), \qquad \sum_{j=1}^{\infty} q_j(x_t) = 1,$$

Nonparametric model: intuition

- Conditional expectation

$$E(y_t|x_t, H_t) = \sum_{j=1}^{\infty} q_j(x_t) E(y_t|x_t, \mu_j, B_j, H_t).$$

- Nonparametric beta

$$b_t^x = \frac{\partial E(y_t|x_t, H_t)}{\partial x_t}.$$

  - For elliptical distributions: $b_t^x = \beta_t$
  - More general distributions: b_t^x is a nonlinear function of factors x_t
  - This corresponds to a nonlinear pricing kernel (Dittmar 2002).

MGARCH-DPM Model

Extending Jensen and Maheu (2013) to asymmetric VD-GARCH, $r_t = (y_t, x_t')'$,

$$
\begin{aligned}
r_t & |\phi_t, H_t \sim N(\xi_t, H_t^{1/2}\Lambda_t(H_t^{1/2})'),\ t = 1, ..., T \\
\phi_t & \equiv \{\xi_t, \Lambda_t\}|G \sim G, \\
G| & \alpha, G_0 \sim DP(\alpha, G_0), \\
G_0 & \equiv N(\mu_0, D) \times \mathcal{W}^{-1}(B_0, \nu_0), \\
H_t & = \Gamma_0 + \Gamma_1 \odot (r_{t-1} - \eta)(r_{t-1} - \eta)' + \Gamma_2 \odot H_{t-1}.
\end{aligned}
$$

- $DP(\alpha, G_0)$ is a Dirichlet process
- G_0 is the base measure; $E[G(A)] = G_0(A)$
- α is the precision parameter: $\text{Var}(G(A)) = G_0(A)(1 - G_0(A))/(1 + \alpha)$

Stick breaking representation

$$p(r_t|\mu, B, W, H_t) = \sum_{j=1}^{\infty} \omega_j N(r_t|\mu_j, H_t^{1/2} B_j (H_t^{1/2})'),$$

$$\omega_1 = v_1,\ \omega_j = v_j \prod_{l=1}^{j-1} (1 - v_l),\ j > 1,$$

$$v_j \overset{iid}{\sim} \text{Beta}(1, \alpha),$$

$$\mu_j \overset{iid}{\sim} N(\mu_0, D),\quad B_j \overset{iid}{\sim} \mathcal{W}^{-1}(B_0, \nu_0),$$

Special cases

- Gaussian: $\alpha \to 0$ as $\omega_1 = 1$, $\omega_j = 0, \forall j > 1$ and $B_1 = I$.
- Student-t: $\mu_j = \mu$, $\forall j$ and $\alpha \to \infty$, since $G \to G_0$,

Slice sampling

$$\text{target: } f(r_t|W,\Theta) = \sum_{j=1}^{\infty} w_j N(r_t|\mu_j, H_t^{1/2} B_j (H_t^{1/2})').$$

Define u_t such that the joint density of (r_t, u_t) is given by

$$f(r_t, u_t|W,\Theta) = \sum_{j=1}^{\infty} \mathbf{1}(u_t < \omega_j) N(r_t|\mu_j, (H_t^{1/2})' B_j H_t^{1/2}).$$

Let $s_t = j$ if $r_t \sim N(\mu_j, H_t^{1/2} B_j (H_t^{1/2})')$. Then the full likelihood is

$$p(r_{1:T}, U, S|W,\Theta) = \Pi_{t=1}^{T} 1(u_t < w_{s_t}) N(r_t|\mu_{s_t}, (H_t^{1/2}) B_{s_t} (H_t^{1/2})')$$

The joint posterior is then

$$p(W_K) \sum_{j=1}^{K} g_0(\mu_j, B_j) \prod_{t=1}^{T} 1(u_t < w_{s_t}) N(r_t|\mu_{s_t}, (H_t^{1/2}) B_{s_t} (H_t^{1/2})')$$

where K is such that $\sum_{j=1}^{K} w_j > 1 - \min\{u_t\}$.

MCMC Steps

The posterior sampler delivers the set of variables

$$\{(\mu_j^{(g)}, B_j^{(g)}), v_j^{(g)}, j = 1, ..., K^{(g)}\}, \{S_t^{(g)}, u_t^{(g)}, H_t^{(g)}, t = 1, ...T\}, g = 1, \ldots, M.$$

The sampling steps are:

1. $\pi(\mu_j, B_j | r_{1:T}, S) \propto g_0(\mu_j, B_j) \prod_{\{t: s_t = j\}} N(r_t | \mu_j, H_t^{1/2} B_j (H_t^{1/2})'),\ j = 1, \ldots, K$
2. $\pi(v_j | S) \propto \text{Beta}(1 + \Sigma_{t=1}^T \mathbf{1}(s_t = j), \alpha + \Sigma_{t=1}^T \mathbf{1}(s_t > j)).$
3. $\pi(u_t | W, S) \propto \mathbf{1}(0 < u_t < w_{s_t}),\ t = 1, \ldots, T.$
4. Find K such that $\sum_{j=1}^K w_j > 1 - \min\{u_t\}$.
5. $p(S_t = j | r_{1:T}) \propto \mathbf{1}(w_j > u_t) N(r_t | \mu_j, H_t^{1/2} B_j (H_t^{1/2})'), j = 1, ..., K$
6. $p(\Gamma | \mu, B, S, r_{1:T}, H_{1:T}) \propto p(\Gamma) \prod_{t=1}^T N(r_t | \mu_{S_t}, H_t^{1/2} B_{S_t} (H_t^{1/2})')$

Nonparametric conditional distribution

At each iteration $g = 1, ..., M$ of the algorithm, a draw of $G|r_{1:T}$, can be written as

$$G^{(g)} = \sum_{j=1}^{K^{(g)}} \omega_j^{(g)} \delta_{\theta_j^{(g)}} + \left(1 - \sum_{j=1}^{K^{(g)}} \omega_j^{(g)}\right) G_0(\theta),$$

where $\theta_j^{(g)} = (\mu_j^{(g)}, B_j^{(g)})$ and $\delta_{\theta_j^{(g)}}$ is a mass point at $\theta_j^{(g)}$.

$$p(\mathbf{y}_t, \mathbf{x}_t | r_{1:T}, G^{(g)}) = \sum_{j=1}^{K^{(g)}} \omega_j^{(g)} f(\mathbf{y}_t, \mathbf{x}_t | \theta_j^{(g)}) + \left(1 - \sum_{j=1}^{K^{(g)}} \omega_j^{(g)}\right) \int f(\mathbf{y}_t, \mathbf{x}_t | \theta) G_0(\theta) d\theta,$$

where $f(\mathbf{y}_t, \mathbf{x}_t | \theta)$ is the multivariate normal density.

Nonparametric conditional distribution

$$p(\mathbf{y}_t|\mathbf{x}_t, r_{1:T}, G^{(g)}) = \frac{p(\mathbf{y}_t, \mathbf{x}_t|r_{1:T}, G^{(g)})}{p(\mathbf{x}_t|r_{1:T}, G^{(g)})}$$

$$= \frac{p(\mathbf{y}_t, \mathbf{x}_t|r_{1:T}, G^{(g)})}{\sum_{j=1}^{K^{(g)}} \omega_j^{(g)} f_2(\mathbf{x}_t|\theta_j^{(g)}) + \left(1 - \sum_{j=1}^{K^{(g)}} \omega_j^{(g)}\right) \int f_2(\mathbf{x}_t|\theta) G_0(\theta) d\theta}$$

$$= \sum_{j=1}^{K^{(g)}} q_j^{(g)}(\mathbf{x}_t) f(\mathbf{y}_t|\mathbf{x}_t, \theta_j^{(g)}) + \left(1 - \sum_{j=1}^{K^{(g)}} q_j^{(g)}(\mathbf{x}_t)\right) f(\mathbf{y}_t|\mathbf{x}_t, G_0),$$

where

$$q_j^{(g)}(\mathbf{x}_t) = \frac{\omega_j^{(g)} f_2(\mathbf{x}_t|\theta_j^{(g)})}{\sum_{j=1}^{K^{(g)}} \omega_j^{(g)} f_2(\mathbf{x}_t|\theta_j^{(g)}) + \left(1 - \sum_{j=1}^{K^{(g)}} \omega_j^{(g)}\right) \int f_2(\mathbf{x}_t|\theta) G_0(\theta) d\theta}$$

and $f_2(\mathbf{x}_t|\theta_j^{(g)})$ is the marginal (normal) density of $\mathbf{x}_t$ and $f(\mathbf{y}_t|\mathbf{x}_t, G_0)$ is the conditional distribution

Nonparametric conditional mean and beta

An estimate of the unknown conditional distribution is obtained as

$$p(\mathbf{y}_t|\mathbf{x}_t, r_{1:T}) \approx \frac{1}{M}\sum_{g=1}^{M} p(\mathbf{y}_t|\mathbf{x}_t, r_{1:T}, G^{(g)}).$$

Conditional mean

$$E(\mathbf{y}_t|\mathbf{x}_t, r_{1:T}, G^{(g)}) = \sum_{j=1}^{K^{(g)}} q_j^{(g)}(\mathbf{x}_t)[\mu_{j,1}^{(g)} + \beta_{jt}^{(g)}(\mathbf{x}_t - \mu_{j,2}^{(g)})] +$$

$$\left(1 - \sum_{j=1}^{K^{(g)}} q_j^{(g)}(\mathbf{x}_t)\right) \frac{\int [\mu_1 + \beta_t(\mathbf{x}_t - \mu_2)] N(\mathbf{x}_t|\mu_2, (H_t^{(g)^{1/2}} B H_t^{(g)^{1/2'}})_{22}) p(\mu, B) d\mu dB}{\int N(\mathbf{x}_t|\mu_2, (H_t^{(g)^{1/2}} B H_t^{(g)^{1/2'}})_{22}) p(\mu, B) d\mu dB}.$$

Nonparametric conditional mean and beta

Integrating all parameter and distributional uncertainty results in an estimate of the predictive conditional mean as

$$E(\mathbf{y}_t|\mathbf{x}_t, r_{1:T}) \approx \frac{1}{M}\sum_{g=1}^{M} E(\mathbf{y}_t|\mathbf{x}_t, r_{1:T}, G^{(g)}).$$

The nonparametric beta is the derivative of this conditional expectation of $\mathbf{y}_t$ given $\mathbf{x}_t$, with respect to $\mathbf{x}_t$. This is,

$$b_t^m(\mathbf{x}_t) = \frac{\partial E(\mathbf{y}_t|\mathbf{x}_t, r_{1:T})}{\partial \mathbf{x}_t}\Big|_{x_t=\mathbf{r}_{m,t}}.$$

Application

- Daily data from (Jan 3, 2000 to Dec 31 2013), 3521 observations
- The value-weighted index from CRSP as market return.
- Individual stocks: excess returns on IBM, GE, XOM, AMGN, and BAC.
- r_t^{SMB} (size), r_t^{HML} (value), r_t^{MOM} (momentum) from Kenneth French's website
- Priors: $\mu_j \sim N(0, 0.1)$, $B_j \sim IW(5I, 8)$, $E[B_j] = I$, $\alpha \sim G(0.1, 0.3)$
- Remaining MGARCH parameters $NID(0, 100)$, $\nu \sim U(2, 100)$.
- $M = 15,000$ after $10,000$ burn-in.

Model Comparison: one factor (market) model

$$m(r_{L:T}|r_{1:L-1}, \mathcal{M}) = \Pi_{t=L}^{T} p(r_t|r_{1:t-1}, \mathcal{M})$$

	log-predictive likelihood				
Model	IBM	GE	XOM	AMGN	BAC
MGARCH-DPM	**−983.27**	**−964.99**	**−875.47**	**−1140.12**	**−1473.11**
BEKK-DPM	−1039.88	−1023.37	−951.14	−1226.54	−1526.39
MGARCH-t	−1353.67	−1369.03	−1300.21	−1571.32	−1684.72
	log-Bayes factors				
MGARCH-DPM vs MGARCH-t	370.40	404.04	424.74	431.20	211.61
MGARCH-DPM vs BEKK-DPM	56.61	58.37	75.67	86.42	53.28

Table: This table reports the log-predictive likelihood for the bivariate MGARCH-DPM, BEKK-DPM and MGARCH-t and log-Bayes factors, for the last 500 observations, from 2012/03/12 to 2013/12/31. Bold entries denote the largest log-predictive likelihood for each asset. Bivariate data are daily excess market returns coupled with excess returns on IBM, GE, XOM, AMGN, and BAC from 2000/01/03 to 2013/12/31.

$$p(r_t|r_{1:t-1}, \text{MGARCH-t}) \approx \frac{1}{M}\sum_{g=1}^{M} t(r_t|\mu^{(g)}, H_t^{(g)}, \nu^{(g)}),$$

$$p(r_t|r_{1:t-1}, \text{MGARCH-DPM}) \approx \frac{1}{M}\sum_{g=1}^{M} N(r_t|\mu_{s_t^{(g)}}^{(g)}, H_t^{(g)^{1/2}} B_{s_t^{(g)}}^{(g)} H_t^{(g)^{1/2'}}).$$

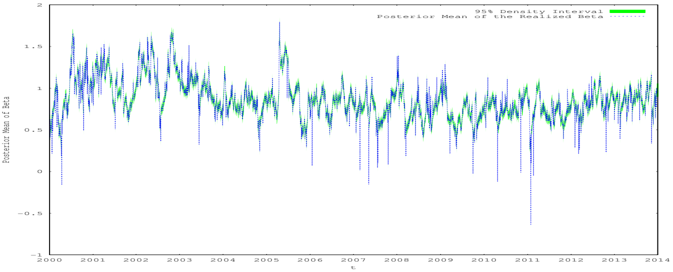
Estimates: IBM

IBM	MGARCH-DPM		MGARCH-t	
Parameter	Post. Mean	95% DI	Post. Mean	95% DI
γ_{01}	0.102	(0.055, 0.146)	0.023	(0.015, 0.037)
γ_{02}	−0.043	(−0.081, 0.003)	−0.042	(−0.053, −0.034)
γ_{03}	0.020	(0.001, 0.053)	0.020	(0.002, 0.048)
γ_{11}	0.247	(0.199, 0.307)	0.150	(0.144, 0.160)
γ_{12}	0.267	(0.232, 0.313)	0.224	(0.210, 0.233)
γ_{21}	0.971	(0.965, 0.977)	0.975	(0.971, 0.977)
γ_{22}	0.953	(0.945, 0.961)	0.955	(0.951, 0.961)
μ_1			0.025	(0.016, 0.046)
μ_2			0.041	(0.022, 0.074)
ν			5.37	(5.01, 5.54)
c	5.6	(3.00, 11.0)		
α	0.571	(0.070, 1.61)		
η_1	0.570	(0.349, 0.714)	0.807	(0.776, 0.864)
η_2	0.533	(0.434, 0.618)	0.507	(0.451, 0.644)

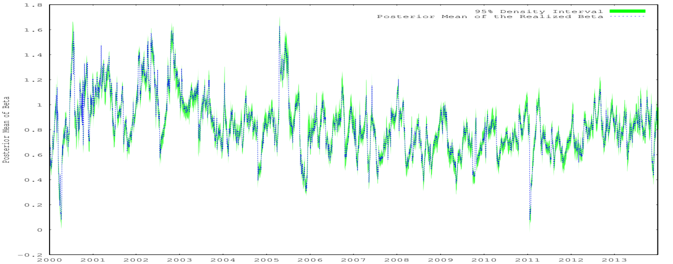
$$
\begin{aligned}
r_t|\phi_t, H_t &\sim N(\xi_t, H_t^{1/2}\Lambda_t(H_t^{1/2})'),\ t = 1, ..., T \\
\phi_t &\equiv \{\xi_t, \Lambda_t\}|G \overset{iid}{\sim} G \\
G|\alpha, G_0 &\sim DP(\alpha, G_0) \\
H_t &= \Gamma_0 + \Gamma_1 \odot (r_{t-1} - \eta)(r_{t-1} - \eta)' + \Gamma_2 \odot H_{t-1} \\
\Gamma_0 &= \Gamma_0^{1/2}(\Gamma_0^{1/2})', \gamma_0 = \text{vech}(\Gamma_0^{1/2}), \Gamma_1 = \gamma_1\gamma_1', \Gamma_2 = \gamma_2\gamma_2'.
\end{aligned}
$$

Beta over time: IBM

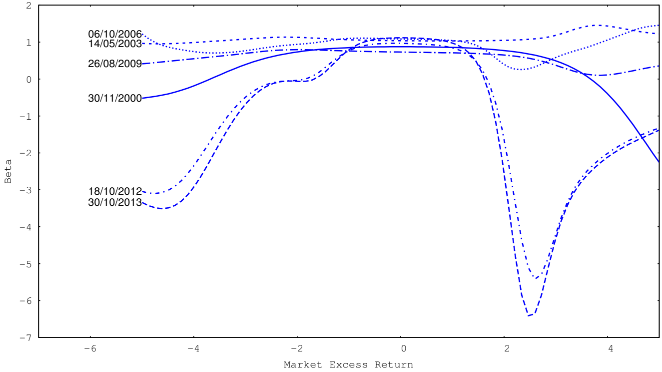
MGARCH-DPM



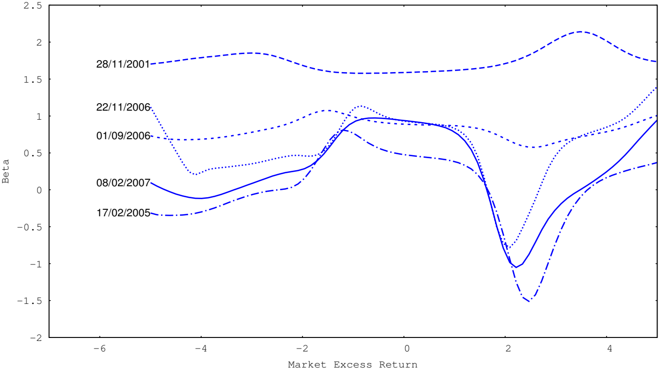
MGARCH-t



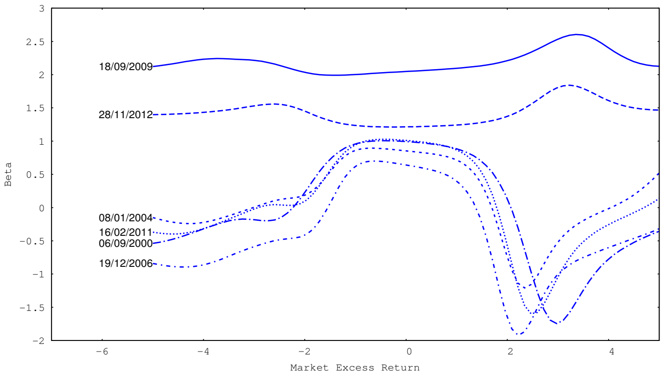
Nonparametric beta: IBM



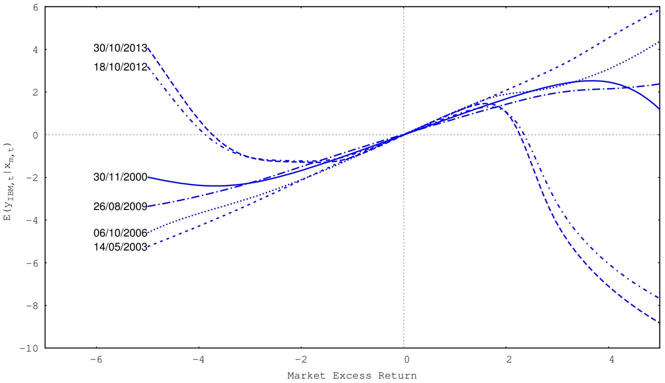
Nonparametric beta: Exxon



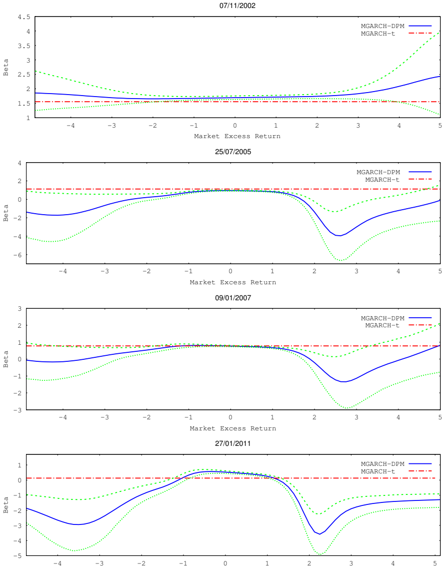
Nonparametric beta: GE



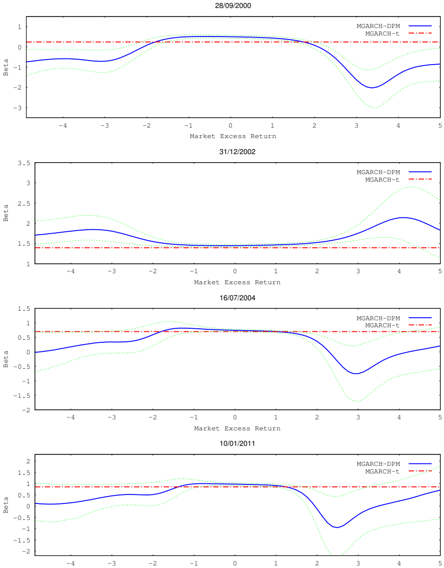
Nonparametric expected return: IBM



Nonparametric beta: IBM

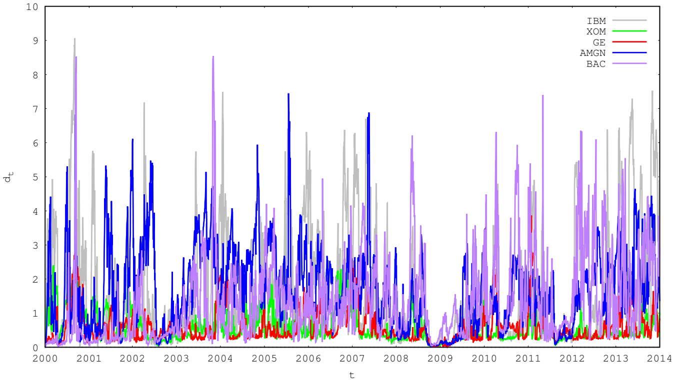


Nonparametric beta: GE

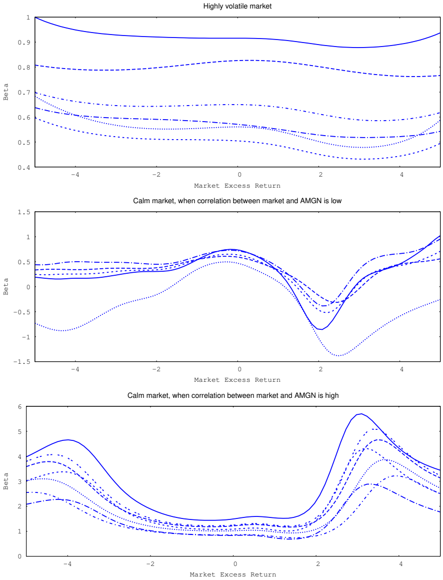


Range of nonparametric beta as function of market

$$d_t = \max_{\mathbf{x}_t} b_t^m(\mathbf{x}_t) - \min_{\mathbf{x}_t} b_t^m(\mathbf{x}_t).$$



Nonparametric beta over different market conditions: AMGN

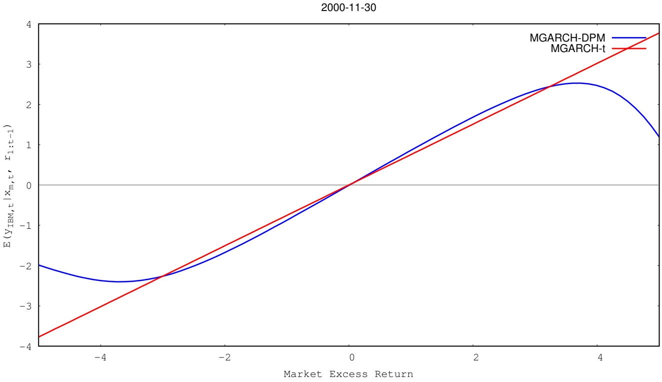


Summary

- Highly volatile market: individual stock's conditional beta is less affected by unexpected shocks in the market return.
- Calm market:
  - $\rho_{i,m}$ is low: an unexpected shock lowers the conditional beta (Inverse U)
  - $\rho_{i,m}$ is high: an unexpected shock increases the conditional beta (U shape)

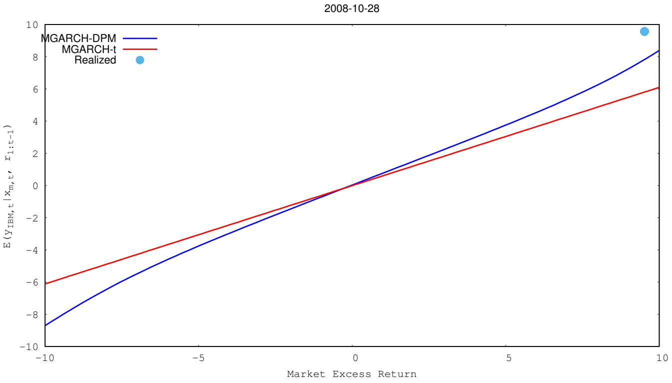
Applications

Conditional expected return for Nov. 30/2000.



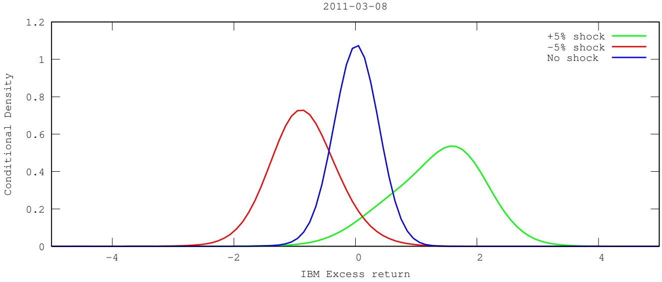
Applications

market return: 9.77%
IBM return 9.77%



All dates with with absolute market return greater than 6%:
MGARCH-DPM RMSFE=8.131,
MGARCH-t RMSFE=8.394.

IBM's predictive conditional density given big market shock.



Value-at-risk	
-5% market shocks:	1.805%
0% market shock:	0.652%
+5% market shock:	0.050%

Selecting the number of factors

Problem: a model with q factors results in $q+1$ assets to model jointly.

1. marginal predictive likelihoods: $p(y_t|x_{1,1:t-1})$ and $p(y_t|x_{1,1:t-1},x_{2,1:t-1})$ are compared for 1 versus 2 factors. These are derived by integrating out the factors from the bivariate and trivariate density respectively.
2. conditional predictive likelihoods: one-factor model against the two-factor model with $p(y_t|y_{1:t-1},x_{1,1:t})$ and $p(y_t|y_{1:t-1},x_{1,1:t},x_{2,1:t})$ for the out-of-sample observations.

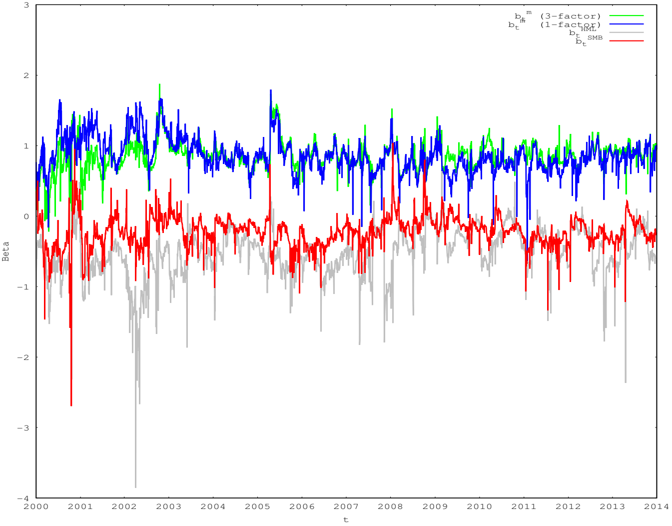
Conditional predictive likelihood given different factors

$$\sum_{t=L}^{T} \log p(y_t | y_{1:t-1}, x_{1,1:t}, x_{2,1:t})$$

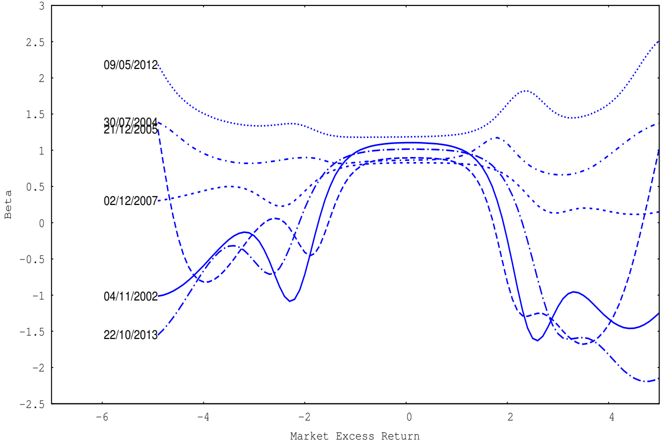
	Log of the Conditional Predictive Likelihood		
Stock	1-factor model	3-factor model	4-factor model
IBM	-651.397	$-\mathbf{549.866}$	-637.261
GE	-603.154	$-\mathbf{498.385}$	-605.745
BAC	$-\mathbf{585.941}$	-597.852	-602.425
XOM	-672.418	$-\mathbf{670.437}$	-678.152
AMGN	-635.289	$-\mathbf{615.284}$	-642.158

Table: This table reports the log of the conditional predictive likelihood for MGARCH-DPM model, for the last 500 observations, from 2012/03/12 to 2013/12/31. Bold entries denote the largest value in each row. Data are daily excess market, HML, SMB returns, and the momentum factor coupled with excess returns on IBM, BAC, XOM, GE, and AMGN from 2000/01/03 to 2013/12/31.

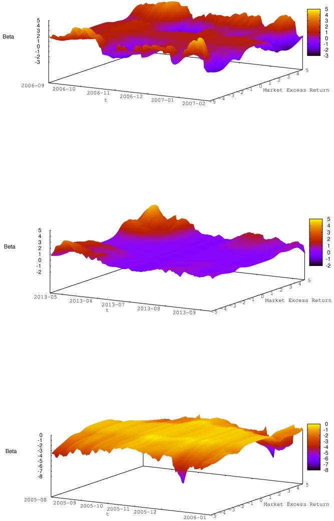
IBM: posterior mean of b_t^m (3-factor), b_t^{SMB}, and b_t^{HML}



IBM: b_t^m, 3-factor MGARCH-DPM model.



IBM: b_t^m (top panel), b_t^{SMB} (middle panel), and b_t^{HML}



Conclusion

- Extend conditional density pricing to semiparametric setting.
- Conditional mean and beta depend nonlinearly on factors.
- Nonparametric beta often strongly depends on the market.
- Nonlinear beta robust to number of factors
- Other factor sensitivities are a nonlinear function of factors.
- Support for a nonlinear three factor specification.