Deep Neural Networks for Estimation and Inference

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Introduction & Outline

1. **Context: Nonparametric Estimation**

- Goal in this paper is to study a standard problem with a "new" tool
- How does deep learning do in a problem we understand well
- We not addressing: high dimensionality, adaptivity, double-descent, ...

2. **Main Results**

- Nonasymptotic high-probability bounds and implied rates
- "Industry standard" setup: multi-layer perceptron, ReLU activation
- Generic results for other architectures

3. **Deep Learning In Economics** (if time)

- Structured models for individual heterogeneity
- Structured network to match
- Semiparametric inference

Deep Feed-Forward Neural Networks:

Problem Set Up

Nonparametric Problem

General regression-type problem

- Outcome Y, d covariates $\boldsymbol{X} \in \mathbb{R}^d$
- Target is $f_\star = \arg\min \mathbb{E}\left[\ell\left(f, Y, \boldsymbol{X}\right)\right]$
- Any loss function such that:
  1. Lipschitz: $\left|\ell(f, y, \boldsymbol{x}) - \ell(g, y, \boldsymbol{x})\right| \leq C_\ell \left|f(\boldsymbol{x}) - g(\boldsymbol{x})\right|$
  2. Curvature: $c_1 \mathbb{E}\left[(f - f_\star)^2\right] \leq \mathbb{E}[\ell(f, Y, \boldsymbol{X})] - \mathbb{E}[\ell(f_\star, Y, \boldsymbol{X})] \leq c_2 \mathbb{E}\left[(f - f_\star)^2\right]$
  - Includes least squares, logistic, poisson, ...

DNN estimator

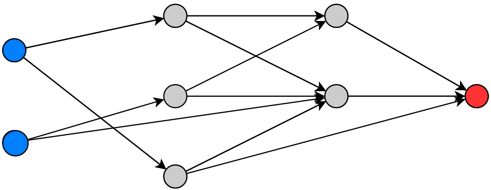
$$\widehat{f}_{\text{DNN}} := \underset{f_{\boldsymbol{\theta}} \in \mathcal{F}_{\text{DNN}}}{\arg\min} \sum_{i=1}^{n} \ell\left(f, y_i, \boldsymbol{x}_i\right), \qquad \text{e.g.} \quad \ell\left(f, y_i, \boldsymbol{x}_i\right) = \frac{1}{2}(y - f(\boldsymbol{x}))^2$$

- No optimization issues here
- Sometimes helpful to think of a linear sieve, but with learned basis: $\widehat{f}(\boldsymbol{x}) = \hat{\boldsymbol{p}}(\boldsymbol{x})'\hat{\boldsymbol{\gamma}}$

Feedforward Neural Networks

A set of units:

- $d = \dim(\boldsymbol{X})$ input units
- One output unit, Y
- U hidden units between



Units are arranged into layers:

- According to a directed, acyclic graph
- Unit is in layer l if it has a predecessor in $l-1$ and none for any $l' \geq l$
- For parameters $\boldsymbol{w}_{h,l}$ and $b_{h,l}$, unit h in layer l computes $\tilde{x}_{h,l+1} = \sigma(\tilde{\boldsymbol{x}}_l' \boldsymbol{w}_{h,l} + b_{h,l})$
  $\hookrightarrow$ layer l returns $\tilde{\boldsymbol{x}}_{l+1} = (\tilde{x}_{1,l+1}, \ldots, \tilde{x}_{H_l,l+1})$
- Dimension of $\tilde{\boldsymbol{x}}_l = H_l =$ *width*
- Number of layers $= L =$ *depth*
- Final layer outputs appropriate $\widehat{f}_{\text{DNN}}(\boldsymbol{x})$, e.g., for LS $\widehat{f} = \tilde{\boldsymbol{x}}_L(\boldsymbol{x})' \boldsymbol{w}_L + b_L$

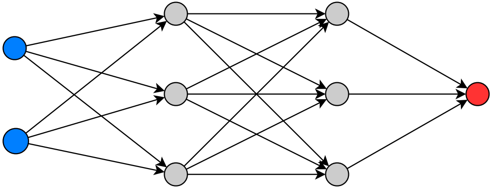
Computation

- Layer by layer, back-propagation implements chain rule
- $\boldsymbol{\theta} =$ all *weights* and *"biases"* $\{(\boldsymbol{w}_{h,l}', b_{h,l})\}$; W total parameters

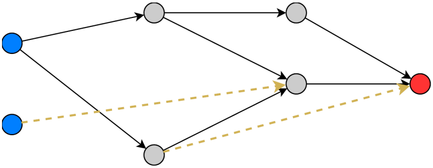
Architecture Choice 1: Multi-Layer Perceptron

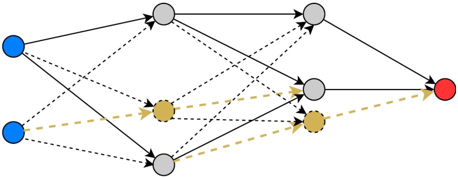
MLP a.k.a. Fully-connected feedforward network

- Fully connected to *adjacent* layers
- No hyperlinks
- Not necessarily “rectangle”
- Now common practice
  (though many others exist)



Embed any feedforward network in an MLP:

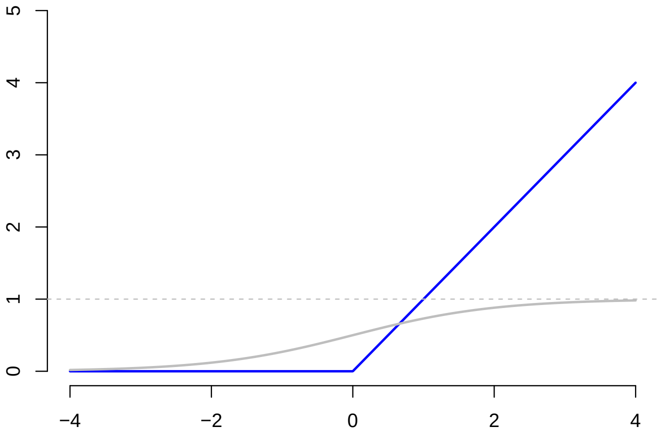




Architecture Choice 2: Rectified Linear Unit Activation Function

The activation function for each node: $\sigma(z) = \max(0, z)$

- Used at each node: $(\tilde{\boldsymbol{x}}_l' \boldsymbol{w}_l + b_l) \mapsto \sigma(\tilde{\boldsymbol{x}}_l' \boldsymbol{w}_l + b_l)$
- The ReLU activation is ubiquitous, replacing sigmoid-type
- Better optimization properties, does not get stuck in zero-gradient areas
- Conjecture: same results hold for any piecewise linear activation, slight modifications



Architecture Choice 3: Unbounded Weights

Parameters of the DNN are "slopes" $\boldsymbol{w}_{h,l}$ and "intercepts" $b_{h,l}$

- Each node $\tilde{\boldsymbol{x}}_l \mapsto (\tilde{\boldsymbol{x}}_l' \boldsymbol{w}_{h,l} + b_{h,l}) \mapsto \sigma(\tilde{\boldsymbol{x}}_l' \boldsymbol{w}_{h,l} + b_{h,l})$
- Common in theory to impost a bound on all parameters:

$$\max_{l \leq L} \max_{h \leq H_l} \|\boldsymbol{w}_{h,l}\|_\infty \vee |b_{h,l}| \leq B$$

- The reason: bounded weights make complexity arguments easier

Practically this seems innocuous

- Because empirically weights are often "small"
- And certainly optimizers don't return $\boldsymbol{w}_{h,l} = \infty$

But actually:

- Bounds cause corner solutions and other computational problems
- How do you set the bound?
- Is this some type of regularization?

What about in theory?

Architecture Choice 3: Unbounded Weights

For intuition, go back to a linear regression model:

$$y_i = x_i\beta + \varepsilon_i$$

The analogue of bounded weights in a NN is a bound on $|\hat{\beta}|$

Seems innocuous: **all** theory and **all** practice assume $|\beta|$ bounded

- Theory: $\mathbb{E}\left[\hat{\beta}_{\text{OLS}} \mid \boldsymbol{X}_n\right] = \beta$
- Practice: How many papers have a table with $\hat{\beta} = \infty$?

The argument goes like this:

1. If $|\beta| < \infty$ (which we all agree on), then there is some $B > 0$ such that $|\beta| < B$
2. The exact constant B doesn't matter, because I can always rescale:

$$y_i = x_i\beta + \varepsilon_i = y_i = \left(x_i B\right)\frac{\beta}{B} + \varepsilon_i$$

3. Therefore, without loss of generality, I restrict estimation to $\left|\hat{\beta}\right| \leq 1$

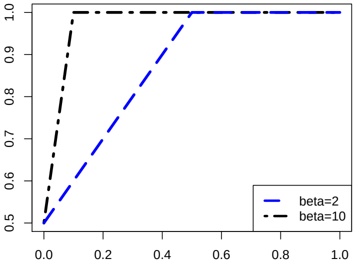
Wait, what?

Architecture Choice 3: Unbounded Weights

Same thing for **neural networks**: instead of *rescaling*, we do a *copying* trick

Simple example:

$$f_\star = \frac{\text{ReLU}(\beta x + 1) - \text{ReLU}(\beta x - 1)}{2}$$



With unbounded weights

1. 2 hidden units for any β
2. $\sqrt{1/n}$ rate for any β

With bounded weights ≤ 1

1. 2β units required
2. $\sqrt{\beta/n}$ rate

If $|\beta| < B$, then "asymptotically" this **copying** makes no difference

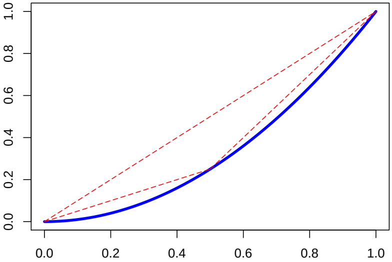
Architecture Choice 3: Unbounded Weights

For nonparametrics, the copying happens on the ideal approximating network

ReLU universal approximation idea:

1. Composition of ReLU = piecewise linear
2. Piecewise linear $\approx$ any polynomial
3. Smooth fcn $\approx$ Taylor series = polynomial

$\Rightarrow$ ReLU $\approx$ any smooth fcn (Yarotsky)



Now argue just like before

1. If $f_\star$ is smooth (say Hölder p) then $\max_{k \le p} \left\| f_\star^{(k)} \right\|_\infty < B$, some B
2. Therefore with bounded weights ≤ 1, need $2Bp =$ "$O(1)$" ReLU units to $\approx f_\star$

There's a **big** difference between $|\beta| < B$ for **some** B vs a **specified, known** B

Nonasymptotic High Probability Bounds

and

Implied Convergence Rates

Main Results

Generic Architecture

- Let $\mathcal{F}_{\text{DNN}}$ be a generic class of feed-forward DNNs with ReLU activation
- Depth $= L$, total parameters $= W$
- Assume $f_\star$ is bounded. Define the bias as $\epsilon_{\text{DNN}} := \inf_{f \in \mathcal{F}_{\text{DNN}}} \|f - f_\star\|_\infty$

Then we have the general result:

With probability at least $1 - e^{-\gamma}$:

$$\mathbb{E}_n\left[(\widehat{f}_{\text{DNN}} - f_\star)^2\right] \leq C\left(\frac{WL\log W}{n}\log n + \frac{\log\log n + \gamma}{n} + \epsilon_{\text{DNN}}^2\right)$$

Comments

- Nonasymptotic, high-probability bounds. Convergence rates follow immediately.
- Final bound depends on the architecture and the assumed function space

Main Results

Multi-Layer Perceptrons and **Smooth Functions**

- Matching standard practice: MLP + ReLU
- For MLP: $W = H_n^2 L_n$, $H_n =$ common width order
- Standard smoothness assumption: $f_\star$ is p-smooth

A leading, important case of the first result is then

$$\mathbb{E}_n\left[(\widehat{f}_{\mathrm{MLP}} - f_\star)^2\right] = O_{\mathbb{P}}\left(\frac{H_n^2 L_n^2 \log(H_n^2 L_n)}{n} \log n + \epsilon_{\mathrm{MLP}}^2\right)$$

$$\ldots\text{at best} = O_{\mathbb{P}}\left(n^{-\frac{p}{p+d}} \log^8 n\right)$$

Comments

- Best rate uses $H_n \asymp n^{\frac{d}{2(p+d)}} \log^2 n$ and $L_n \asymp \log n$
- Fast enough for semiparametrics ... but not optimal $(n^{-\mathbf{2}p/(\mathbf{2}p+d)})$
- Relies on best-known approximation results for MLP-ReLU: $\epsilon_{\mathrm{MLP}} = \epsilon_{\mathrm{MLP}}(H_n, L_n)$
- Loose bounds? Or really suboptimal?

Main Results

Two other interesting results

1. Optimal rate
   - A special, cooked-up architecture delivers $n^{-\frac{2p}{2p+d}}$, i.e. Stone's bound
   - Only of theoretical interest; architecture not practical

2. Fixed-width
   - An MLP, with $H = 2d + 10 \not\to \infty$ and $L_n \asymp n^{\frac{d}{2(2+d)}}$, can obtain $n^{-\frac{2}{2+d}}$
   - Most **different** from 1990s results
   - Interesting in transfer learning, but limited by rate (i.e. $p = 1$)

Comment: future results?

- New approximations (bounds on bias ϵ_{DNN}) can yield new rates immediately
- E.g.: fixed-width for $p > 1$ $\rightarrow$ Corollary 2 immediately sharpens
- E.g.: Sharper ϵ_{MLP} for MLP-ReLU $\rightarrow$ Faster rate on previous slide

Proof Intuition

Start with usual decomposition:

- Truth: $f_\star$
- Estimate: $\hat{f}$
- Best Approximation: $f_n \in \mathcal{F}_{\text{DNN}}$, $\arg\min \|f - f_\star\|_\infty$
- Usual bias-variance:

$$\|\hat{f} - f_\star\|^2_{L_2(X)} \lesssim \underbrace{(\mathbb{E} - \mathbb{E}_n)\left[\ell(\hat{f}, \boldsymbol{z}) - \ell(f_\star, \boldsymbol{z})\right]}_{\text{Empirical Process}} + \underbrace{\mathbb{E}_n\left[\ell(f_n, \boldsymbol{z}) - \ell(f_\star, \boldsymbol{z})\right]}_{\text{Bias}}$$

Key motivation in the proof

- Control the empirical process without bounding the weights

Proof Intuition: Empirical Process

Empirical Process Term

- Employ *localization* techniques, e.g. Bartlett, Bousquet, Mendelson (2005)
- Sharp VC dimension bounds for ReLU networks: Bartlett et al (2017)

Rademacher Complexity

$$R_n\mathcal{F} := \sup_{f\in\mathcal{F}} \frac{1}{n}\sum_{i=1}^{n} \eta_i f(x_i).$$

- Rademacher draws, $\eta_i = \pm 1$
- Intuitively measures how flexible the function class is for predicting random signs
- Have good control of *Empirical Rademacher Complexity* $\mathbb{E}_\eta[R_n\mathcal{F}]$ (via VC dim)

Key Idea: Recursive Improvement

- Suppose we knew $\|\hat{f} - f_\star\|^2_{L_2(X)} \le {r_0}^2$
- Let $\mathcal{G} := \{g = \ell(f, \boldsymbol{z}) - \ell(f_\star, \boldsymbol{z}) : f \in \mathcal{F}_{\text{DNN}}, \|f - f_\star\|^2_{L_2(X)} \le {r_0}^2\}$
- Use this to tighten the bound

Proof Intuition: Empirical Process

Step (i) Symmetrization: w.p. $\geq 1 - 2e^{-\gamma}$

$$(\mathbb{E} - \mathbb{E}_n)\left[\ell(\hat{f}, \boldsymbol{z}) - \ell(f_\star, \boldsymbol{z})\right] \lesssim \mathbb{E}_\eta R_n \mathcal{G} + \sqrt{\frac{r_0{}^2 \gamma}{n}} + \frac{\gamma}{n}$$

Step (ii) Complexity Bound:

$$\mathbb{E}_\eta R_n \mathcal{G} \lesssim r_0 \sqrt{\frac{\mathrm{Pdim}(\mathcal{F}_{\text{DNN}})}{n} \log n} \lesssim r_0 \sqrt{\frac{WL \log(W)}{n} \log n}$$

Step (iii) Improve the Initial Bound:

$$\begin{aligned}
\|\hat{f} - f_\star\|_{L_2(X)}^2 &\lesssim (\mathbb{E} - \mathbb{E}_n)\left[\ell(\hat{f}, \boldsymbol{z}) - \ell(f_\star, \boldsymbol{z})\right] + \mathbb{E}_n\left[\ell(f_n, \boldsymbol{z}) - \ell(f_\star, \boldsymbol{z})\right] \\
&\lesssim r_0 \cdot \left(\sqrt{\frac{WL \log W}{n} \log n} + \sqrt{\frac{\gamma}{n}}\right) + \epsilon_n^2 + \epsilon_n \sqrt{\frac{\gamma}{n}} + \frac{\gamma}{n} \\
&= r_0 o(r_0) \ll r_0{}^2 = \text{initial bound}
\end{aligned}$$

Step (iv) Stopping point:

This trick keeps working if $r_0 > \sqrt{\frac{WL \log W}{n} \log n}$ $\square$

Deep Learning In Economics:

Individual Heterogeneity

The Problem

1. We are interested in some **economic model** with heterogeneous effects:

$$\mathcal{L}\big(\text{data}\ ,\ \beta_i\big)$$

   However:
   - Only with long panels $\hat{\beta}_i \approx \beta_i$
   - Estimates may not conform to theory
   - And what about new individuals?

2. Instead we **balance** structure and flexibility, using covariates:

$$\mathcal{L}\big(\text{data}\ ,\ \beta(\boldsymbol{x}_i)\big)$$

   - Intuitively the same as using interaction terms, but $\beta(\boldsymbol{x})$ is a flexible function

3. Specifically we use **deep neural networks** to capture the heterogeneity:

$$\mathcal{L}\big(\text{data}\ ,\ \widehat{\beta}_{\text{DNN}}(\boldsymbol{x}_i)\big)$$

Framework for Heterogeneity

1. Model $\mathbb{E}[Y \mid \boldsymbol{x}, \boldsymbol{t}] = G\big(\alpha(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x})'\boldsymbol{t}\big)$

- Special case of $\mathcal{L}\big(\{y_i, \boldsymbol{t}_i, \boldsymbol{x}_i\}\ ,\ \boldsymbol{\beta}(\boldsymbol{x}_i)\big)$, just for today
- Old history in statistics, a.k.a. varying/functional/smooth coefficient model
- But here our motivation is heterogeneity + structure
  - \+ Interpretable model
  - \+ Economically meaningful
  - \+ Respects economic principles
  - \+ Fully flexible heterogeneity

Compare to:

- Classical parametric model: $\mathbb{E}[Y \mid \boldsymbol{x}, \boldsymbol{t}] = G\big(\alpha + \boldsymbol{\beta}'\boldsymbol{t} + \boldsymbol{\gamma}'\boldsymbol{x}\big)$
  - \+ Interpretable, meaningful
  - – Limited heterogeneity, no flexibility

- Fully nonparametric, naive ML model: $\mathbb{E}[Y \mid \boldsymbol{x}, \boldsymbol{t}] = G\big(u(\boldsymbol{t}, \boldsymbol{x})\big)$
  - \+ Fully heterogeneous and flexible
  - – Mostly uninterpretable, may not make economic sense
  - – Cannot recover second-stage objects

Why Deep Learning?

The **model** imposes economic structure, respects economic theory, ...

$$\mathbb{E}[Y \mid \boldsymbol{x}, \boldsymbol{t}] = G\big(\alpha(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x})'\boldsymbol{t}\big)$$

We need an estimator that is "**structurally compatible**"

- The model structure is baked directly into the estimation
- Global restriction, easy to impose

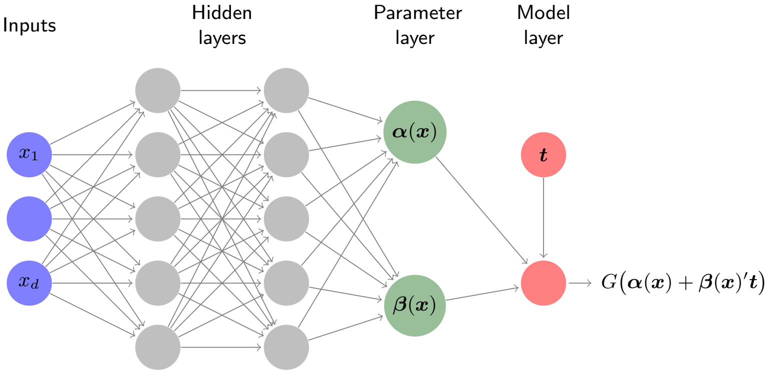
DNNS are an ideal method

- **Both** structurally compatible and capable of tackling modern problems
- Economic structure helps in implementing ML

Why Deep Learning?

Big picture point: implementing ML in economics, why are we ignoring economics?

- Avoid pure prediction: $\hat{y} = \hat{f}(\boldsymbol{x}, t)$, instead learn $\alpha(\boldsymbol{x})$ and $\boldsymbol{\beta}(\boldsymbol{x})$ **jointly**
- Point is not to adapt to structure
- We design our **novel architecture** to model heterogeneity, enforce structure:



Main Results

Key Results

- Only $d = \dim(\boldsymbol{x})$ affects the rate, i.e. dimension of heterogeneity
  - Naive ML approach have slow rate: $\dim(\boldsymbol{x}) + \dim(\boldsymbol{t})$
- Discrete variables handled seamlessly **and** don't affect rate
  - Former uses the same architecture idea, and is more novel; latter is standard

Main Rate Theorem

- Building on FLM1
- Say $\alpha(\cdot)$, $\beta(\cdot)$ are p-smooth in the d_c continuous covariates
- Set width $H_n \asymp n^{\frac{d_c}{2(p+d_c)}} \log^2 n$ and depth $L_n \asymp \log n$

$$\boxed{\mathbb{E}_n\left[(\widehat{\beta}_{\text{DNN}} - \beta)^2\right] \;=\; O_{\mathbb{P}}\left(n^{-\frac{p}{p+d_c}} \log^8 n\right)}$$

Framework for Heterogeneity

1. Model $\mathbb{E}[Y \mid \boldsymbol{x}, \boldsymbol{t}] = G\big(\alpha(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x})'\boldsymbol{t}\big)$

2. Inference Target $\theta_0 = \mathbb{E}\big[H(\boldsymbol{X}, \alpha(\boldsymbol{X}), \boldsymbol{\beta}(\boldsymbol{X}), \boldsymbol{t}^*)\big]$

Researcher chooses H function:

- Vectors are allowed
- So are implicit functions
- Not possible without structure
- Only require you to compute $\nabla H = (\partial H/\partial\alpha,\ \partial H/\partial\boldsymbol{\beta})'$

1+2 simultaneously cover many economically interesting contexts:

- Recovering existing results: ATE, partially linear model, ...
- Delivering **new** applications: choice models, average partial effects, continuous/multiple treatments, production functions, count data, Berry logit, ...
- Idea extends to IV, multinomial choice, systems, ...

Framework for Heterogeneity

1. Model $\mathbb{E}[Y \mid \boldsymbol{x}, \boldsymbol{t}] = G\big(\alpha(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x})'\boldsymbol{t}\big)$

2. Inference Target $\theta_0 = \mathbb{E}\big[H(\boldsymbol{X}, \alpha(\boldsymbol{X}), \boldsymbol{\beta}(\boldsymbol{X}), \boldsymbol{t}^*)\big]$

Main **theoretical result** of this section:

- **Single** influence function calculation, $\psi - \theta_0$, (our model structure + Newey 1994)

$$\psi = H(\alpha, \boldsymbol{\beta}, \boldsymbol{x}) \; + \; \nabla H(\alpha, \boldsymbol{\beta}, \boldsymbol{x})' \boldsymbol{\Lambda}(\boldsymbol{x})^{-1}(1, \boldsymbol{t})' \Big(y - G\big(\alpha(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x})'\boldsymbol{t}\big)\Big)$$

- One crucial piece is $\boldsymbol{\Lambda}(\boldsymbol{x}) = \mathrm{var}[\dot{G}^{1/2}\boldsymbol{T}|\boldsymbol{x}]$, $\dot{G} = \partial G(u)/\partial u$
  - Intuition: generalization of the inverse propensity part of the IF
  - Implementation: Estimate in observational data, but **compute** in experiments
  - For semiparametric inference in general: small denominators are a bear
- Inference then follows standard methods, e.g. cross-fitting (Chernozhukov et al 2018)

Seeing it All in Action:

What is Advertising Worth?

How does advertising content affect purchasing?

Bertrand, Karlan, Mullainathan, Shafir, Zinman (QJE, 2010)

- Advertising for shortish-term loans in South Africa
- $Y = \{0, 1\}$ = Applied for a loan
- $\boldsymbol{T}$ = 12 ad characteristics, **randomly** assigned (based on $\boldsymbol{x}$)
  - Interest rate: directly compute value
  - Other qualities (photos, tables, uses) can be valued
- $\boldsymbol{X}$ = 11 individual characteristics, some discrete
- Original questions: does advertising content matter? How much?
- Original results: marginal effects from probit

Applying the Framework

1. Structural model: $\mathbb{E}[Y \mid \boldsymbol{x}, \boldsymbol{t}] = \text{logit}\big(\alpha(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x})'\boldsymbol{t}\big)$
2. Average marginal effects:
$$\theta_0 = \mathbb{E}\left[\frac{\partial \mathbb{E}[Y|\boldsymbol{X}, t^*]}{\partial t}\right] = \mathbb{E}\big[\underbrace{\beta(\boldsymbol{X})G(1-G)}_{H(\alpha,\beta)}\big]$$
3. Estimate $\widehat{\alpha}_{\text{DNN}}(\boldsymbol{x}_i)$, $\widehat{\boldsymbol{\beta}}_{\text{DNN}}(\boldsymbol{x}_i)$
4. Inference via IF
   - Compute $\boldsymbol{\Lambda}(\boldsymbol{x})$: nontrivial, but computable

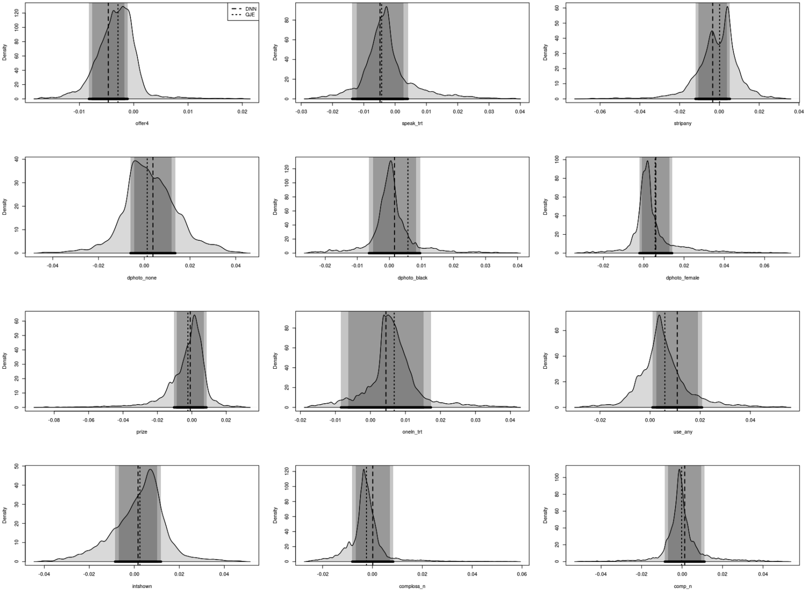
What couldn't we do before?

- Structural model allows for computation of marginal effects
- Deep learning means 11-dimensional nonparametrics feasible
  - **13!** nonparametric functions: $\alpha(\boldsymbol{x}), \boldsymbol{\beta}(\boldsymbol{x})$
- Standard errors rely on novel IF

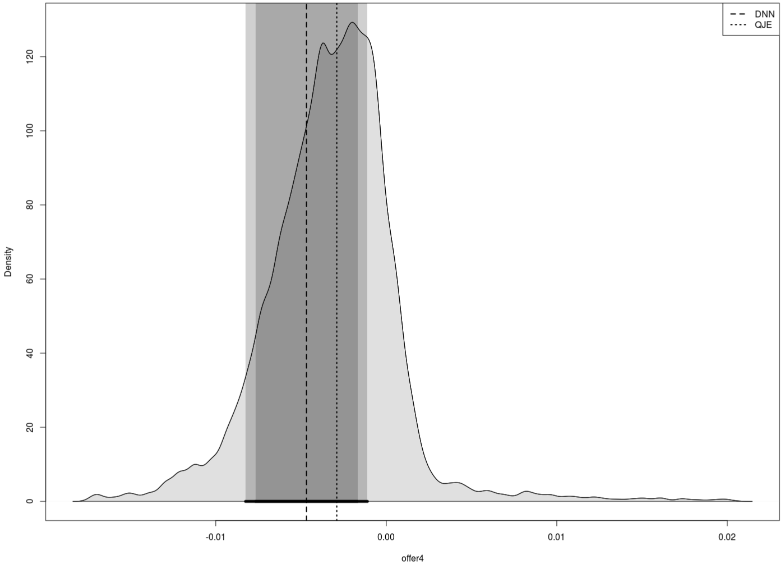
Results Table

Variable	QJE	DNN ME	95% CI		$\Pr(\beta(x) > 0)$	Coef. of Variation
Interest rate offer	-0.0029	-0.0047	-0.0083	-0.0011	0.1337	1.0211
We speak your language	-0.0043	-0.0048	-0.0137	0.0041	0.2533	2.0542
Special rate for you	0.0001	-0.0034	-0.0120	0.0053	0.5001	4.4506
No photo	0.0013	0.0038	-0.0060	0.0136	0.5723	3.4931
Black photo	0.0058	0.0016	-0.0064	0.0096	0.5402	5.1348
Female photo	0.0057	0.0060	-0.0021	0.0141	0.6820	2.3375
Cell phone raffle	-0.0023	-0.0009	-0.0104	0.0085	0.4812	17.0059
Example loan shown	0.0068	0.0044	-0.0084	0.0173	0.8631	1.9379
No loan use mentioned	0.0059	0.0108	0.0009	0.0207	0.7499	1.0936
Interest rate shown	0.0025	0.0017	-0.0085	0.0119	0.6289	7.9903
Loss comparison	-0.0024	0.0001	-0.0081	0.0083	0.2342	89.3606
Competitors rate shown	-0.0002	0.0013	-0.0085	0.0111	0.4107	9.6790

Estimates


DNN
QJE
Density
offer4
speak_trt
stripany
dphoto_none
dphoto_black
dphoto_female
prize
oneln_trt
use_any
intshown
comploss_n
comp_n

Offer Rate Coefficient



Beyond Marginal Effects: Optimal Offers

Some Real Economics

- Assume the firm wishes to maximize profits:

$$\pi_i = \max_{r=\text{rate}} L\left[rG\left(r\right)\right]\left[1 - D\left(r\right)\right]$$

  - $L =$ expected dollar loan amount, normalize to 1 (doesn't impact the rate)
  - $r =$ the interest rate offered
  - $G(r) =$ the probability of acceptance (depends on $\alpha(\boldsymbol{x}_i)$, $\boldsymbol{\beta}(\boldsymbol{x}_i)$)
  - $\left[1 - D\left(r\right)\right] =$ probability of non-default on the loan
(calibrated to match the results in the QJE using a logit kernel)

- Then it is straightforward to show that

$$\frac{\partial \pi}{\partial r} = \left(r\dot{G}\left(r\right)\beta_r + G\left(r\right)\right)\left[1 - D\left(r\right)\right] - rG\left(r\right)\dot{D}\left(r\right)\delta = 0$$

Beyond Marginal Effects: Optimal Offers

- Simplifying:

$$\frac{\partial \pi}{\partial r} = \left(r\left(1 - G\left(r\right)\right)\beta_r + 1\right)\left[1 - D\left(r\right)\right] - r\dot{D}\left(r\right)\delta = 0$$

- And using the logit structure:

$$r = \frac{1 + r\left(1 - G\left(r\right)\right)\beta_r}{D\left(r\right)\delta}$$

- There will a unique fixed point since the denominator of the RHS is decreasing in r for $\beta_r < 0$ and $\delta > 0$
- Therefore:

$$r^* = \frac{1 + r^*\left(1 - G\left(r^*\right)\right)\beta_r}{D\left(r^*\right)\delta}$$

Applying the Framework

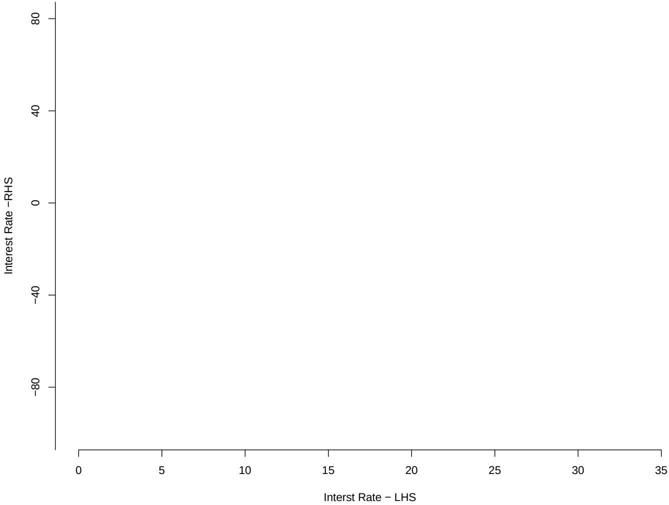
- Even if we don't have it in **closed form**, r^* is a smooth function of $\alpha(\boldsymbol{x}), \boldsymbol{\beta}(\boldsymbol{x})$

$\Rightarrow$ We can do inference on $\theta_0 = \mathbb{E}[H(\alpha, \boldsymbol{\beta}, r^*)]$

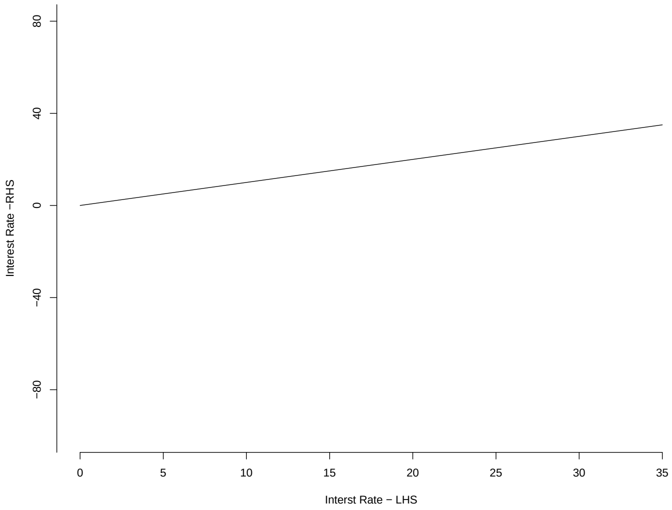
  - derivatives can be calculated via implicit differentiation or numerically: $\dfrac{\partial H}{\partial r^*}\dfrac{\partial r^*}{\partial \beta}$

- Impossible without our results and without exploiting heterogeneity

Optimal Offers



Optimal Offers



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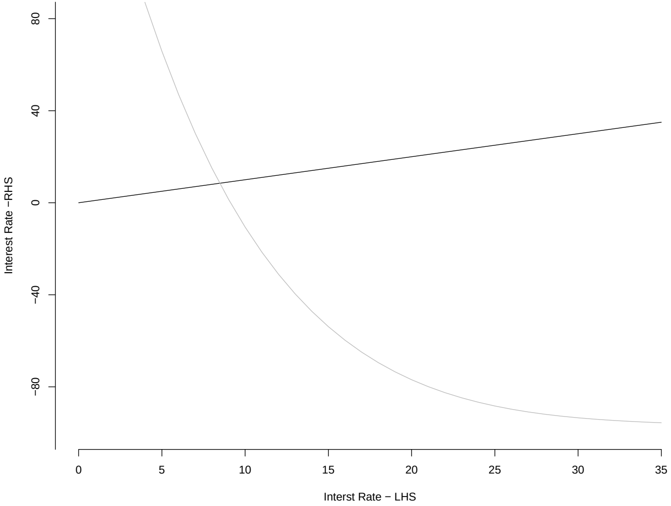

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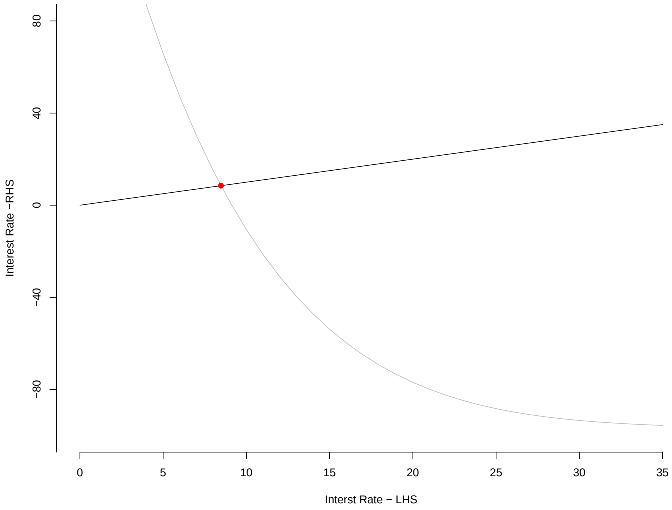

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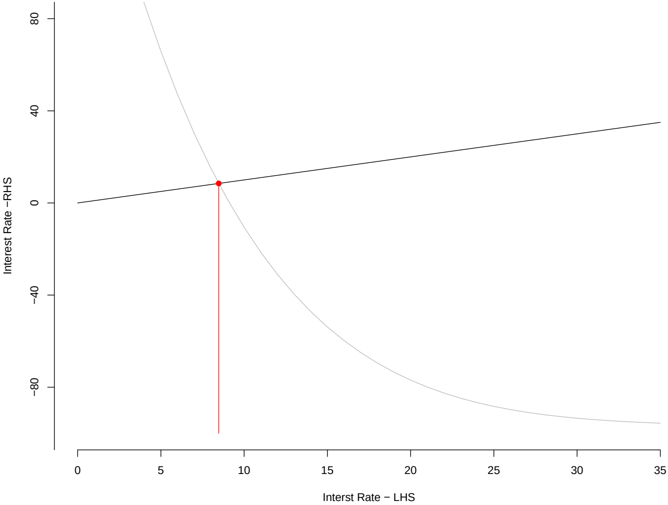

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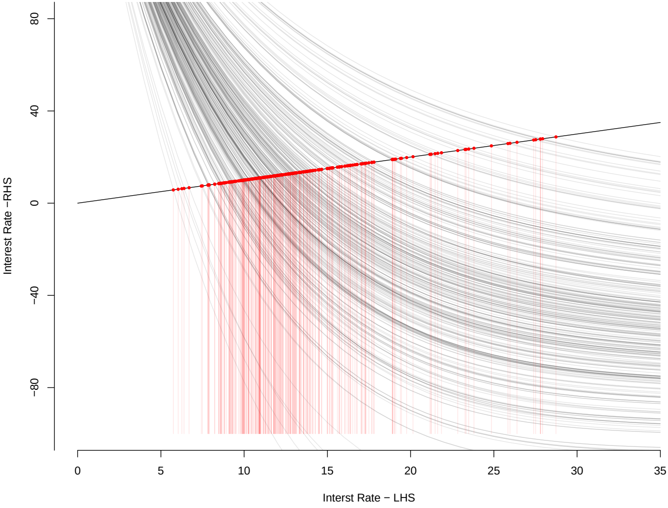

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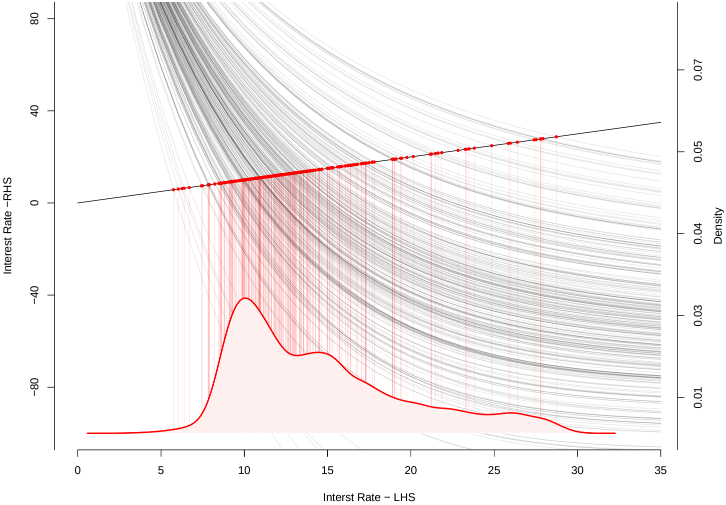

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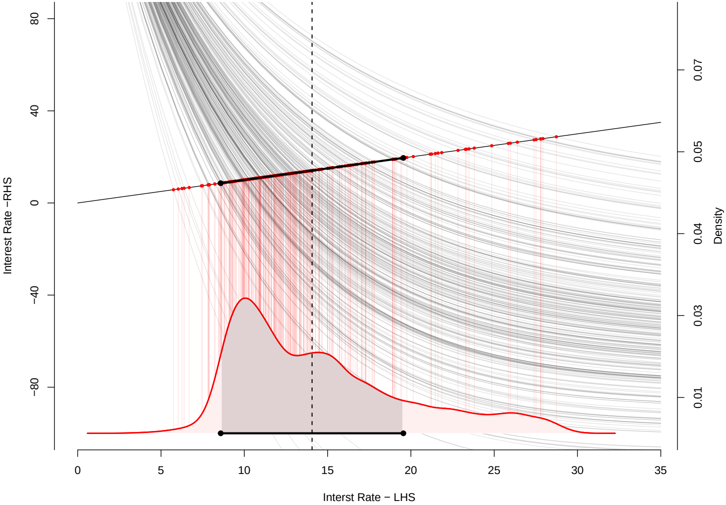

Wrapping Up

Wrapping Up

Deep Learning can have a place in the economics toolbox
Our results are practicable & broadly applicable

Contributions to deep learning itself

- Nonasymptotic bounds for general regression problems
- Matching common practice
  - Fully connected network, ReLU activation, unbounded weights
- Shows good empirical performance

Contribution to economics $\cap$ ML: structured models with heterogeneity

- Wide ranging methodology
- Novel architecture for structured models
- Inference via novel IF
- Big picture point: implementing ML in economics, don't ignore economics

Long way to go

- Computational issues: optimization, tuning, easy to use tools, ...
- Theoretical issues: optimal architectures, depth vs. width, different data types, ...

Thanks!