Reducing Dimensions in a Large TVP-VAR

Eric Eisenstat, University of Queensland
Joshua Chan, Purdue University
Rodney Strachan, University of Queensland

WU 2018

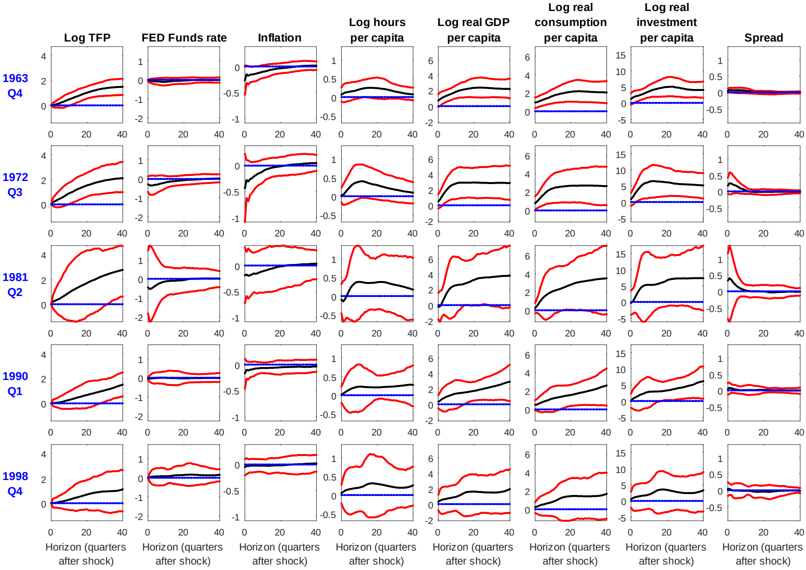
Trends in VAR modelling: Larger and time varying

- Bayesian VARs have been getting larger
  - Improved forecasts (see, e.g., Pirschel and Wolters (2018))
  - Avoid (mitigate) problems with interpreting impulse responses
  - Avoid (mitigate) omitted variable bias
    - Banbura, Giannone and Reichlin (2010), Carriero, Clark and Marcellino (2011), Carriero, Kapetanios and Marcellino (2009), Giannone, Lenza, Momferatou and Onorante (2010), Koop (2011)
- VARs have become more flexible (TVP-VAR)
  - Capture evolution of the economy
    - Cogley and Sargent (2005), Cogley, Morozov and Sargent, (2005), Primiceri (2005), Koop, Leon-Gonzalez and Strachan (2009), Canova and Forero (2012).

Large TVP-VARs

- Large TVP-VARs seem a natural next step
  - A significant impediment to employing larger TVP-VARs is dimension
    - With more variables (N), VAR model dimensions grow at $O\left(N^2\right)$
    - Depending upon the correlation structure of the states, with more variables (N), $TVP - VAR$ model dimensions grow at between $O\left(N^2\right)$ and $O\left(N^4\right)$
  - Overparameterization leads to poor estimation and inference (e.g., wide error bands on impulse response functions)
  - Higher dimensions complicate (e.g., slow) computation

Large VARs and TVP-VARs



Large TVP-VARs

- Strategies to reduce the dimensions but maintain the full structure are an active area of research ... but progress has been slow:
  - Koop and Korobilis (2013) use forgetting factors ($N = 3, 7, 25$)
    - Imposes a tight structure on the autocorrelations of filtered estimates
    - They use the Kalman filter - subject to Sims' critique of Cogley and Sargent (2001)
    - Excellent forecasting performance
    - Does not provide a formal inferential framework
  - Koop and Korobilis (forthcoming) use compressed VARs ($N = 7, 19, 129$)

In this paper

- Develop a TVP-VAR that uses a reduced number of states but preserves (potentially) the full TVP-VAR
- We impose rank reduction on the covariance matrix for the state equations, as suggested by the data
  - In doing so, we generalize the centering and parameter expansion approach of Frühwirth-Schnatter and Wagner (2010)
  - An interpretation of our model specification is that we use a factor structure for the states
- Demonstrate increased precision in estimating time varying parameters

VAR to SVAR

- For an N vector

$$y_t = (y_{1,t}, y_{2,t}, \ldots, y_{N,t})' \quad (N \times 1)$$

the standard TVP-VAR has the form

$$\begin{aligned} y_t &= \mu_t + \Pi_{1,t} y_{t-1} + \Pi_{2,t} y_{t-2} + \nu_t \\ E\left(\nu_t \nu_t'\right) &= \left(A_{0,t}\right)^{-1} \Sigma_t \left(A_{0,t}\right)^{-1\prime} \end{aligned}$$

- The structural form (the TVP-SVAR) measurement equation can be written as

$$A_{0,t} y_t = \mu_t + A_{1,t} y_{t-1} + A_{2,t} y_{t-2} + \varepsilon_t$$

$$\varepsilon_t = A_{0,t} \nu_t \qquad E\left(\varepsilon_t \varepsilon_t'\right) = \Sigma_t$$

Why change from the VAR to SVAR?

- For the TVP-VAR, estimation using a Gibbs sampler follows

$$\Pi_{i,t}|A_{o,t},\Sigma_t$$
$$A_{o,t}|\Pi_{i,t},\Sigma_t$$
$$\Sigma_t|\Pi_{i,t},A_{o,t},$$

  - However, the relationship between $\Pi_{i,t}$ and $A_{o,t}$ implies they are highly correlated - the Markov chain will be less efficient than it could be

- For the TVP-SVAR, estimation using a Gibbs sampler follows

$$A_{j,t}|\Sigma_t$$
$$\Sigma_t|A_{j,t}$$

  - The $A_{j,t}$ are all drawn in one single block leading to a more efficient Markov chain

TVP-(S)VAR models

In structural form the TVP-SVAR measurement equation can be written as

$$y_t = (y_{1,t}, y_{2,t}, \ldots, y_{N,t})' \quad (N \times 1)$$

$$\begin{aligned} A_{0,t} y_t &= \mu_t + A_{1,t} y_{t-1} + A_{2,t} y_{t-2} + \varepsilon_t \\ A_{0,t} &= I - A^*_{0,t} \end{aligned}$$

$$y_t = \mu_t + A^*_{0,t} y_t + A_{1,t} y_{t-1} + A_{2,t} y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \Sigma_t) \qquad \Sigma_t = diag\left(e^{h_{1,t}}, e^{h_{2,t}}, \ldots, e^{h_{N,t}}\right)$$

TVP-SVAR models

The TVP-SVAR can then be written in the more familiar regression form as

$$y_t = \mu_t + A_{0,t}^* y_t + A_{1,t} y_{t-1} + A_{2,t} y_{t-2} + \varepsilon_t$$

or

$$y_t = x_t \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t)$$
$$\alpha_t = \alpha_{t-1} + \eta_t \quad \eta_t \sim N(0, Q)$$

- The $(k \times 1)$ vector α_t has error covariance matrix Q
- The matrix Q is often specified as full or diagonal
- Bubbling away in the background in the literature have been discussions around the form of Q

Consider how state space models produce estimates

$$y_t = x_t \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \Sigma_t\right)$$
$$\alpha_t = \alpha_{t-1} + \eta_t \quad \eta_t \sim N\left(0, Q\right)$$

- With one observation per state, the correlation structure implied by the state equation allows us to estimate α_t
- To see this, stack observations and rewrite the state space model as

$$y = X\alpha + \varepsilon \text{ and } R\alpha = \gamma + \eta$$

such that

$$\alpha \sim N\left(R^{-1}\gamma, V_\alpha\right) \quad V_\alpha = R^{-1}\left(I_T \otimes Q\right)\left(R^{-1}\right)'$$

  - If $Q = I$, then $V_{\alpha,ij} = \min\{i, j\}$.

Consider how state space models produce estimates ...

$$\alpha \sim N\left(R^{-1}\gamma, V_\alpha\right) \quad V_\alpha = R^{-1}\left(I_T \otimes Q\right)\left(R^{-1}\right)'$$

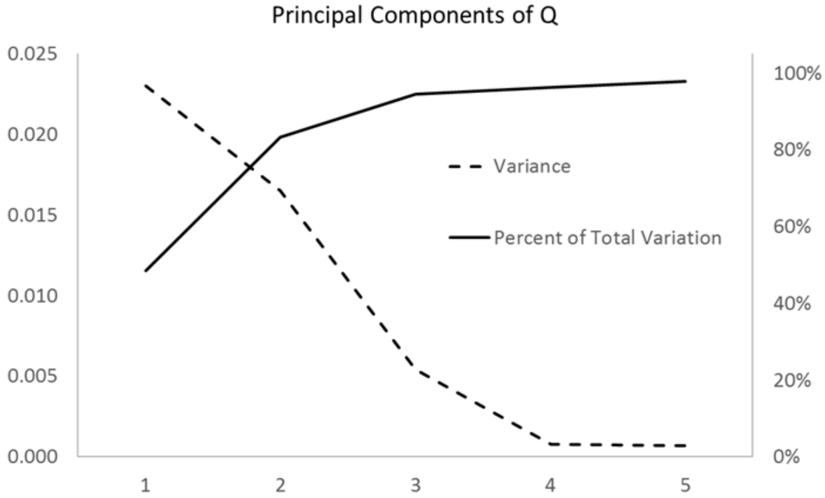
- If all states were independent, i.e., $R = I_T$ and $Q = I_k$ so $V_\alpha = I$, then the states are independent and estimation would be very poor
- Through R and Q, states are allowed to be correlated as V_α is a full psd matrix (as in Cogley and Sargent).
  - Information in the data is shared among all states: the higher the correlation the better the transmission of information
- If V_α has reduced rank because Q has reduced rank, there is perfect correlation, such that information in the data is *perfectly* transmitted among states

An observation by Cogley and Sargent

$$y_t = x_t \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t)$$
$$\alpha_t = \alpha_{t-1} + \eta_t \quad \varepsilon_t \sim N(0, Q)$$

- With $N = 3$ variables and 2 lags, α_t is $(k \times 1)$ where $k = 21$
- Cogley and Sargent use a full PSD matrix for the 21×21 matrix Q
- The states α_t are very highly correlated such that Q looks to have reduced rank

An observation by Cogley and Sargent (cont.)



Reduced rank Q

Take the singular value decomposition of Q :

$$Q = U\Lambda U' \qquad U'U = I_k \text{ and } \Lambda = diag\left(\lambda_1, \ldots, \lambda_k\right)$$
$$U \in O\left(k\right) \equiv \left\{U : U'U = I_k\right\}$$
$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_k \geq 0$$

- If $rank\left(Q\right) = r < k$ then $\lambda_{r+1} = \lambda_{r+2} = \cdots = \lambda_k = 0$
- Partition U and Λ conformably as $U = \left[U_1 \quad U_2\right]$ and

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & 0 \end{bmatrix}$$

then

$$Q = U\Lambda U' = \underset{k\times r}{U_1}\underset{r\times r}{\Lambda_1}\underset{r\times k}{U_1'}$$

The specification

Begin with

$$
\begin{aligned}
y_t &= x_t \alpha_t^* + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\
\alpha_t^* &= \alpha_{t-1}^* + \eta_t \quad \eta_t \sim N(0, Q)
\end{aligned}
$$

- We will assume perfectly correlated states and so Q has reduced rank $r_\alpha < k$
  - For the application in the paper, $k = 570$ and $r_\alpha = 4$
- We will use recentering and parameter expansions to obtain a readily computable form

Recentering

Frühwirth-Schnatter and Wagner (2010) use a scalar state α_t^*

$$y_t = x_t \alpha_t^* + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t)$$
$$\alpha_t^* = \alpha_{t-1}^* + \eta_t \quad \eta_t \sim N(0, \sigma)$$

- Recenter by

$$\alpha_t^* = \alpha + \sqrt{\sigma}\widetilde{\alpha}_t$$

such that

$$\begin{aligned} y_t &= x_t \alpha + x_t \sqrt{\sigma}\widetilde{\alpha}_t + \varepsilon_t \\ \widetilde{\alpha}_t &= \widetilde{\alpha}_{t-1} + \widetilde{z}_t \quad \widetilde{z}_t \sim N(0, 1) \\ \widetilde{\alpha}_0 &= 0 \end{aligned}$$

Generalized Recentering

Begin with

$$y_t = x_t \alpha_t^* + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t)$$
$$\alpha_t^* = \alpha_{t-1}^* + \eta_t \quad \eta_t \sim N(0, Q)$$

- Recenter by (generalising Frühwirth-Schnatter and Wagner (2010))

$$\alpha_t^* = \alpha + Q^{1/2} \widetilde{\alpha}_t$$

such that

$$\begin{aligned} y_t &= x_t \alpha + x_t Q^{1/2} \widetilde{\alpha}_t + \varepsilon_t \\ \widetilde{\alpha}_t &= \widetilde{\alpha}_{t-1} + \widetilde{z}_t \quad \widetilde{z}_t \sim N(0, I_k) \\ \widetilde{\alpha}_0 &= 0 \end{aligned}$$

- If $Q = U_1 \Lambda_1 U_1'$ then we can write $Q^{1/2} = U_1 \Lambda_1^{1/2} U_1'$
- Using $Q^{1/2} = U_1 \Lambda_1^{1/2} U_1'$ rewrite the model as

$$
\begin{aligned}
y_t &= x_t \alpha + x_t U_1 \Lambda_1^{1/2} U_1' \widetilde{\alpha}_t + \varepsilon_t \\
&= x_t \alpha + x_t U_1 \Lambda_1^{1/2} \alpha_t + \varepsilon_t
\end{aligned}
$$

$$
U_1' \widetilde{\alpha}_t = U_1' \widetilde{\alpha}_{t-1} + U_1' \widetilde{z}_t \quad U_1' \widetilde{z}_t \sim N(0, I_{r_\alpha})
$$

$$
\alpha_t = U_1' \widetilde{\alpha}_t \text{ and } z_t = U_1' \widetilde{z}_t
$$

- Then we have

$$
\begin{aligned}
y_t &= x_t \alpha + x_t U_1 \Lambda_1^{1/2} \alpha_t + \varepsilon_t \\
\alpha_t &= \alpha_{t-1} + z_t \quad z_t \sim N(0, I_{r_\alpha})
\end{aligned}
$$

Removing the sign in recentering

Frühwirth-Schnatter and Wagner (2010) use a scalar state α_t^*

$$y_t = x_t \alpha_t^* + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \Sigma_t\right)$$
$$\alpha_t^* = \alpha_{t-1}^* + \eta_t \quad \eta_t \sim N\left(0, \sigma\right)$$

- Recenter by

$$\alpha_t^* = \alpha + \sqrt{\sigma}\widetilde{\alpha}_t$$

- Let $\iota \in \{-1, 1\}$, $\alpha_t = \iota\widetilde{\alpha}_t$ and $a = \iota\sqrt{\sigma}$
- The measurement and state equations become

$$\begin{aligned} y_t &= x_t \alpha + x_t a \alpha_t + \varepsilon_t \\ \alpha_t &= \alpha_{t-1} + z_t \quad z_t \sim N\left(0, 1\right) \\ \alpha_0 &= 0 \end{aligned}$$

Parameter expansion

Where Frühwirth-Schnatter and Wagner introduce ι, we introduce C

- Let $C \in O(r_\alpha)$ such that $C'C = I_{r_\alpha}$ as

$$\begin{aligned} y_t &= x_t\alpha + x_t U_1 \Lambda_1^{1/2} \alpha_t + \varepsilon_t \\ &= x_t\alpha + x_t U_1 \Lambda_1^{1/2} C' C \alpha_t + \varepsilon_t \\ &= x_t\alpha + x_t A \alpha_t + \varepsilon_t \\ C\alpha_t &= C\alpha_{t-1} + Cz_t \quad Cz_t \sim N(0, I_{r_\alpha}) \\ \alpha_t &= \alpha_{t-1} + z_t \quad z_t \sim N(0, I_{r_\alpha}) \end{aligned}$$

where

$$A = U_1 \Lambda_1^{1/2} C' \text{ and } \alpha_t = C\alpha_t$$

A specification for reduced sources of errors model

The final model has a dynamic factor structure

$$\begin{aligned} y_t &= x_t \alpha + x_t A \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\ \alpha_t &= \alpha_{t-1} + z_t \quad z_t \sim N(0, I_{r_\alpha}) \quad \alpha_0 = 0 \end{aligned}$$

$$\alpha_t^* = \alpha + A\alpha_t$$

- This model is fully identified up to orthogonal rotations of A and α_t
- The usual identifying restrictions are not imposed (or required) ensuring order invariance
- Computation is fast and efficient

A fuller specification with stochastic volatility

We allow for stochastic volatility in a standard way

$$
\begin{aligned}
y_t &= x_t \alpha_t^* + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\
\Sigma_t &= diag\left(e^{h_{1,t}^*}, e^{h_{2,t}^*}, \ldots, e^{h_{N,t}^*}\right) \\
h_t^* &= \left(h_{1,t}^*, h_{2,t}^*, \ldots, h_{n,t}^*\right)'
\end{aligned}
$$

- The state equations:

$$
\begin{aligned}
\alpha_t^* &= \alpha_{t-1}^* + \eta_t^* \quad \eta_t^* \sim N(0, Q) \\
h_t^* &= h_{t-1}^* + \nu_t \quad \nu_t \sim N(0, Q_h)
\end{aligned}
$$

- We consider three specifications that reduce dimensions

Specification 1:

- We allow for a small range of correlation structures

$$
\begin{aligned}
y_t &= x_t \alpha_t^* + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\
\alpha_t^* &= \alpha_{t-1}^* + \eta_t \quad \eta_t \sim N(0, Q) \\
h_t^* &= h_{t-1}^* + \nu_t \quad \nu_t \sim N(0, Q_h)
\end{aligned}
$$

- Reduce errors only in the mean equation

$$
\begin{aligned}
y_t &= x_t \alpha + x_t A \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\
\alpha_t &= \alpha_{t-1} + z_t \quad z_t \sim N(0, I_{r_\alpha}) \quad \alpha_0 = 0 \\
h_t^* &= h_{t-1}^* + \nu_t \quad \nu_t \sim N(0, Q_h)
\end{aligned}
$$

- A is $(k \times r_\alpha)$

Specification 2:

- Reduce errors only in the mean and log variance equation

$$
\begin{aligned}
y_t &= x_t\alpha + x_t A_\alpha \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \Sigma_t\right) \\
\alpha_t &= \alpha_{t-1} + z_t \quad z_t \sim N\left(0, I_{r_\alpha}\right) \quad \alpha_0 = 0 \\
h_t^* &= h + Q_h^{1/2}\widetilde{h}_t = h + A_h h_t \\
h_t &= h_{t-1} + z_t^h \quad z_t^h \sim N\left(0, I_{r_h}\right)
\end{aligned}
$$

- A_α is $(k \times r_\alpha)$ and A_h is $(N \times r_h)$

Specification 3:

- Let $\theta_t^* = (\alpha_t^*, h_t^*)'$ have a full covariance matrix

$$\begin{aligned} y_t &= x_t \alpha_t^* + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\ \theta_t^* &= \theta_{t-1}^* + \eta_{\theta,t} \quad \eta_{\theta,t} \sim N(0, Q_\theta) \end{aligned}$$

- Recentering and reducing the rank of Q_θ

$$\begin{aligned} y_t &= x_t \alpha + x_t A_\alpha \alpha_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\ \theta_t^* &= \theta + Q_\theta^{1/2} \widetilde{\theta}_t = \theta + A_\theta \theta_t \\ \theta_t &= \theta_{t-1} + z_t^\theta \quad z_t^\theta \sim N(0, I_{r_\theta}) \\ A_\theta &= \begin{bmatrix} A_\alpha \\ A_h \end{bmatrix} \end{aligned}$$

- A_θ is $(k+N) \times r_\theta$

Priors

- In specification one, the mean equation coefficients $[\alpha \quad A]$ require priors

$$\begin{aligned} y_t &= x_t\alpha + x_t A\alpha_t + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \Sigma_t\right) \\ \alpha_t &= \alpha_{t-1} + z_t \quad z_t \sim N\left(0, I_{r_\alpha}\right) \quad \alpha_0 = 0 \end{aligned}$$

- $a = vec\left(A\right) \sim N\left(0, cI_{(n+k)r}\right)$ which equates to using a Wishart prior for Q for the full rank $(r = k)$ case (generalizing Frühwirth-Schnatter and Wagner (2010))
- For elements of $\alpha = \{\alpha_j\}$ we combine SSVS with Minnesota priors as suggested in Korobilis (2013)

Posterior estimation

- Draws of α, $a_\alpha = vec\left(A_\alpha\right)$, h, $a_h = vec\left(A_h\right)$, α_t and h_t (with states for the mixture approximation) are drawn in the established approach for Specifications 1 and 2. All are (conditionally) Gaussian
- For Specification 3, an accept-reject Metrolpolis-Hastings (ARMH) algorithm (Chan and Strachan (2012)) is used for θ_t

The effect of reducing dimension

Some simple accounting shows the extent of dimension reduction
The figures below are all states and all mean parameters in Specification 1

N	0	1	2	4	$TVP - VAR$
3	27	460	587	841	2730
5	70	755	925	1265	8725
10	265	1685	2050	2780	58450
15	585	2890	3575	4945	220425
20	1030	4370	5500	7760	612775

Based upon a $VAR(2)$ with an intercept and $T = 100$

% reduction in dimension

N	0	1	2	4	$TVP-VAR$
3	99.0%	83.2%	78.5%	69.2%	0%
5	99.2%	91.3%	89.4%	85.5%	0%
10	99.5%	97.1%	96.5%	95.2%	0%
15	99.7%	98.7%	98.4%	97.8%	0%
20	99.8%	99.3%	99.1%	98.7%	0%

Based upon a $VAR(2)$ with an intercept and $T = 100$

Application: the effect of reducing dimension

- Estimate a $TVP - VAR$ with two lags for $N = 2$ variables
  - $k = 11$ states in α_t and $N = 2$ states in h_t
  - $r_\alpha = 0, ..., 11$, $r_h = 1, 2$ and $r_\theta = 1, ..., 13$
- We estimate the standard $TVP - VAR$ and compare to the model with $r_\alpha = 11$ (should be the same) and $r_\alpha = 3$
  - This implies a (mild) reduction in dimension from $2,937$ to $1,144$ (61% reduction)

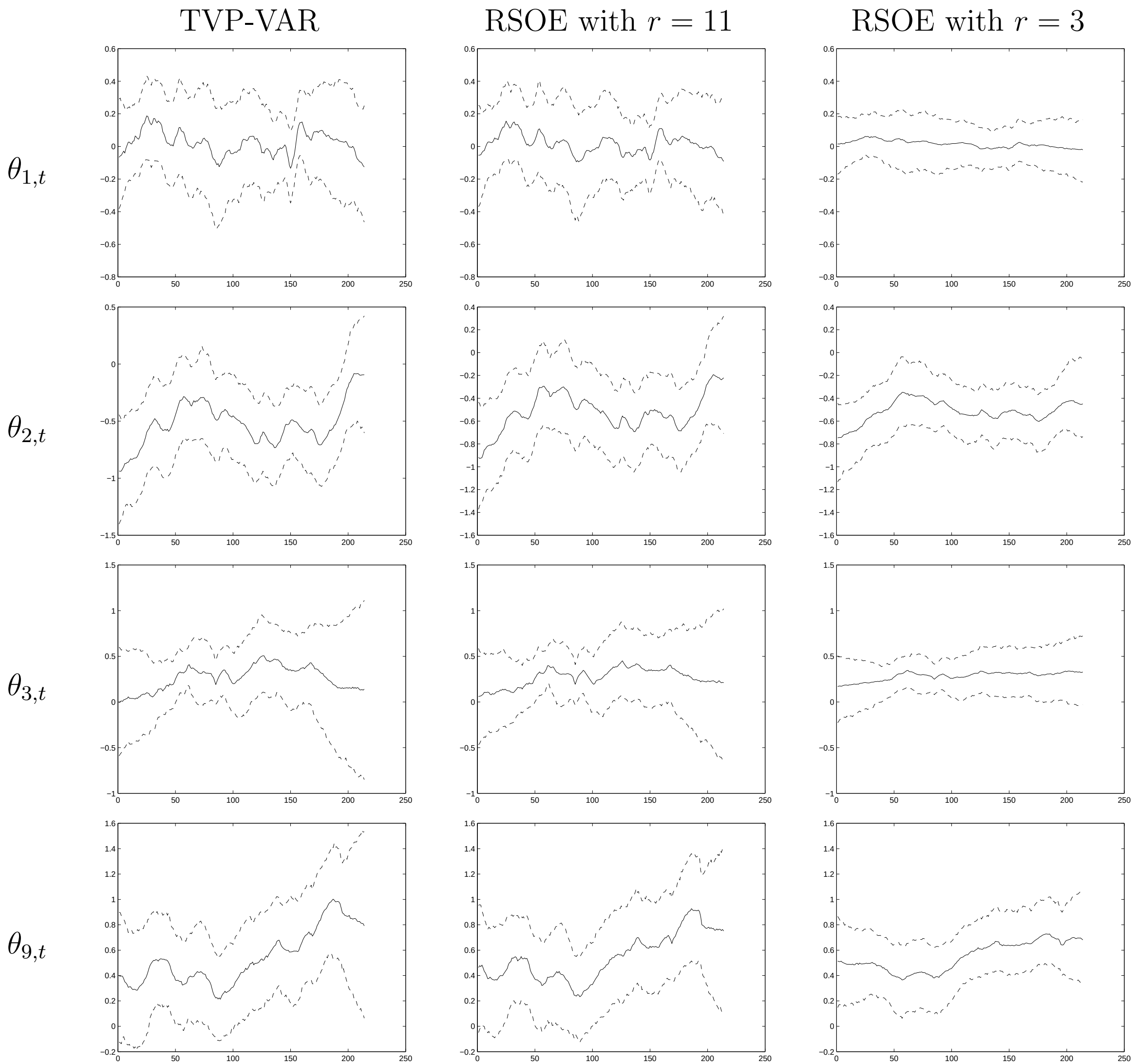


Figure 1: Test example: bivariate (inflation and interest rates) with stationary and demeaned series TVP-VAR ($p = 2$, so $k = 11$).

Application: News and non-news shocks

- Estimate the time-varying effects of surprise productivity and news shocks
- $TVP - SVAR$ with two lags for $N = 15$ variables
  - $k = 570$ parameters in α_t and $N = 15$ parameters in h_t
  - states: $r_\alpha = 0, ..., 570$, $r_h = 0, ..., 15$ and $r_\theta = 0, ..., 585$
- We use Deviance Information Criteria (DIC) to select among the specifications
  - DIC have been shown to perform well for latent variable models, e.g., state space models
- The identification strategy is the same as in Barsky and Sims (2011), with minor modifications for the time-varying parameter context

DICs for models specified with $n = 15$ and various combinations of r_α and r_h. All values are relative to the DIC of the constant coefficient model (i.e. $r_\alpha = r_h = 0$).

3 states			5 states			7 states			10 states		
r_α	r_h	DIC	r_α	r_h	DIC	r_α	r_h	DIC	r_α	r_h	DIC
3	0	-764	5	0	-766	7	0	-742	10	0	-366
2	1	-771	4	1	-816	6	1	-688	8	2	-486
1	2	-711	3	2	-887	**4**	**3**	**-892**	6	4	-697
0	3	-562	2	3	-851	3	4	-888	5	5	-854
			1	4	-756	1	6	-698	4	6	-876
			0	5	-583	0	7	-565	2	8	-800
									0	10	-545
shared		-770	shared		-835	shared		-719	shared		-418

Model selection

- The preferred model with $N = 15$ is Specification 2 has $DIC = -892$ with
  - $r_\alpha = 4$ states driving the mean equation coefficients and
  - $r_h = 3$ states driving the volatilities
  - this implies a dimension reduction from $309,690$ to $4,660$ (98.5%)
- The best Specification 3 (shared states) has $DIC = -835$ with $r_\theta = 5$
  - a dimension reduction of 98.5%.
- Specification 1 is not competitive

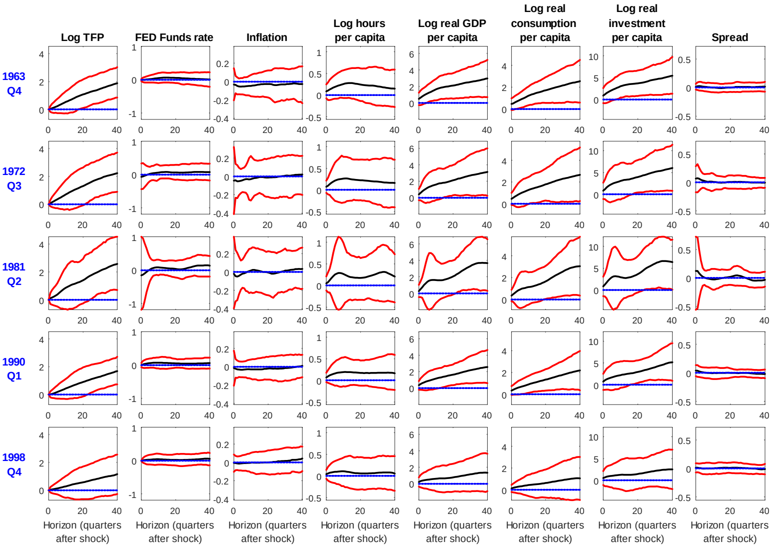


Figure: Impulse-response functions to news shocks in 1963Q4, 1972Q3, 1981Q2, 1990Q1 and 1998Q4 (mean, and 16-84 percentiles of the posterior distribution) for the $n = 15$ variables model.

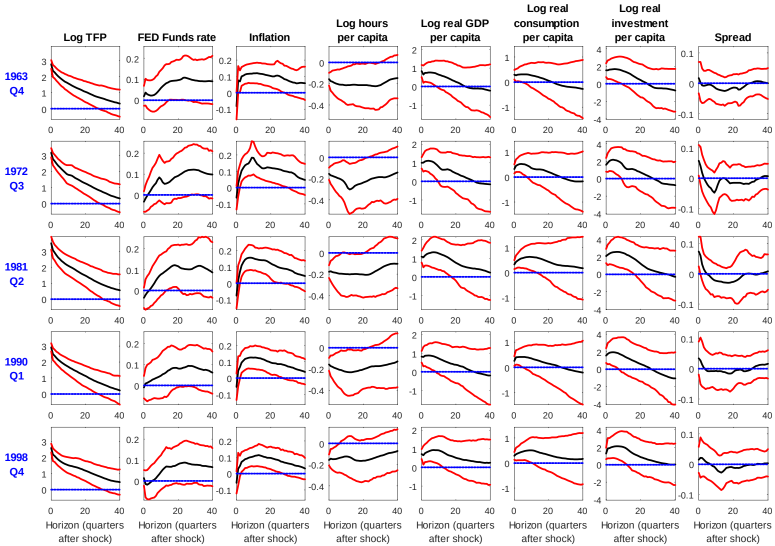


Figure: Impulse-response functions to non-news shocks in 1963Q4, 1972Q3, 1981Q2, 1990Q1 and 1998Q4 (mean, and 16-84 percentiles of the posterior distribution) for the $n = 15$ variables model.

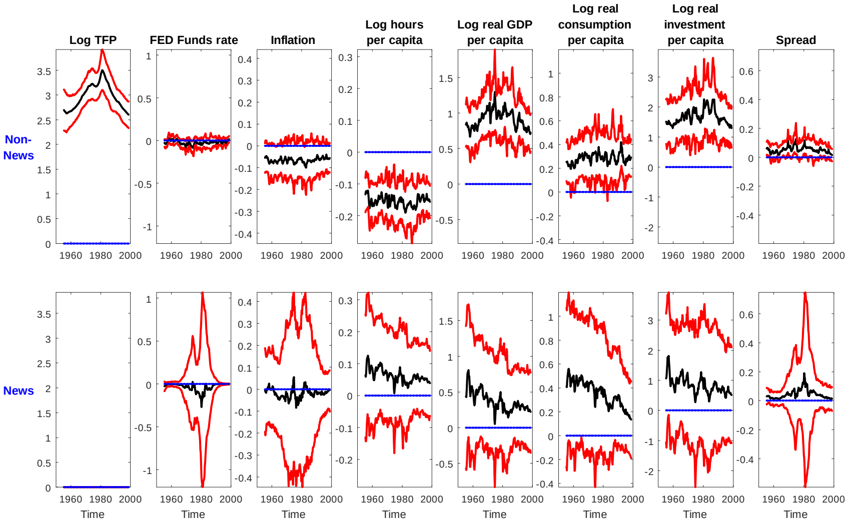


Figure: Time-varying responses to non-news and news shocks on impact (mean, and 16-84 percentiles of the posterior distribution) for the $n = 15$ variables model.

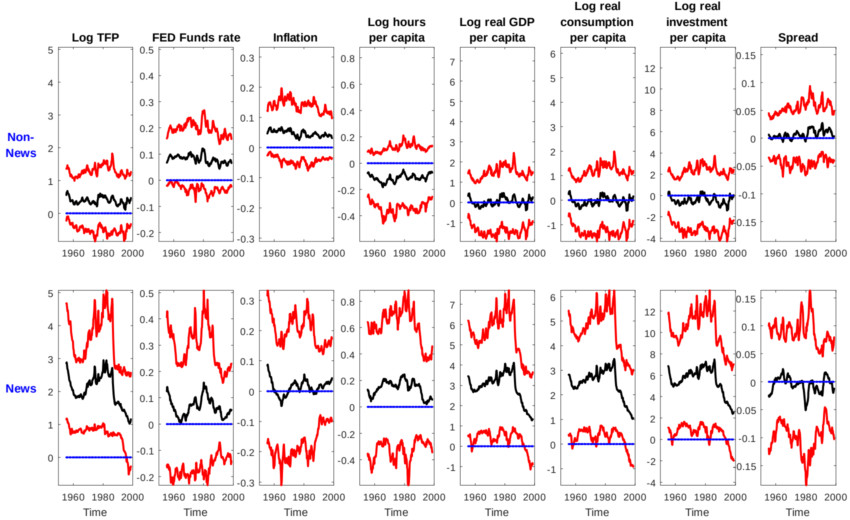


Figure: Time-varying responses to non-news and news shocks at 40 quarters after impact (mean, and 16-84 percentiles of the posterior distribution) for the $n = 15$ variables model.

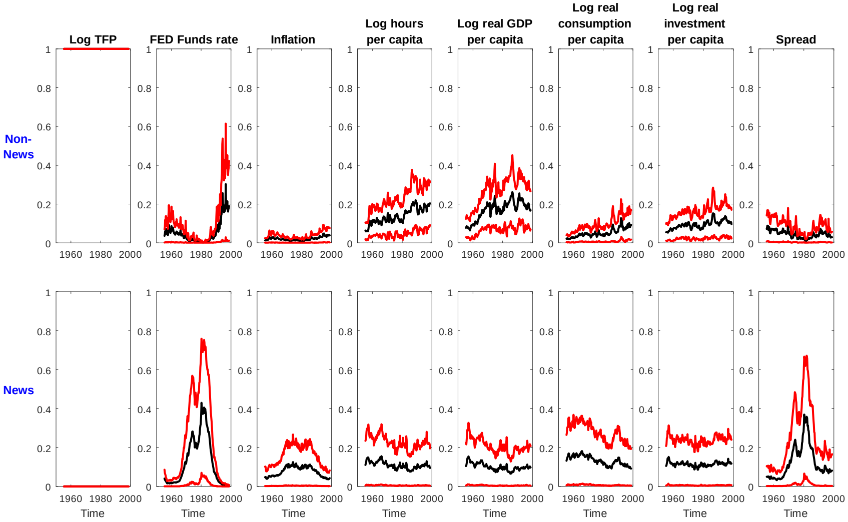


Figure: Time-varying fractions of forecast error variance explained by non-news and news shocks on impact (mean, and 16-84 percentiles of the posterior distribution) for the $n = 15$ variables model.

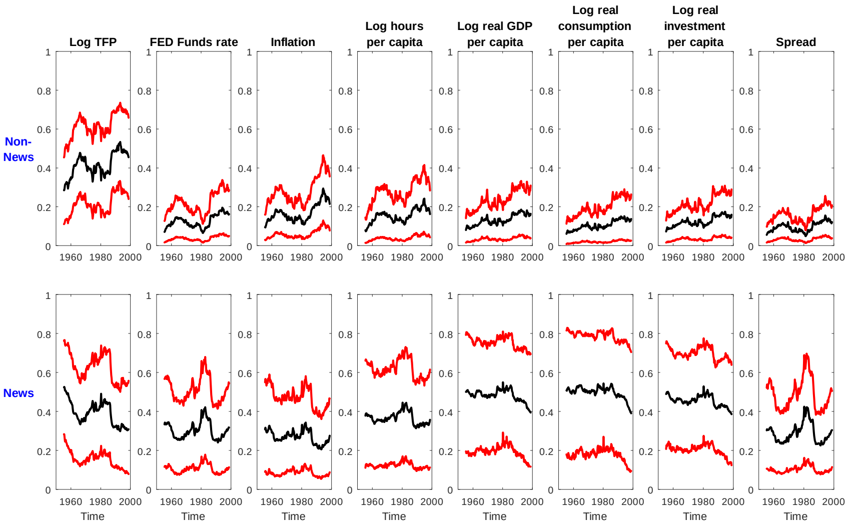


Figure: Time-varying fractions of forecast error variance explained by non-news and news shocks at 40 quarters after impact (mean, and 16-84 percentiles of the posterior distribution) for the $n = 15$ variables model.

Time Variation

- We find evidence of time variation to both news and non-news shocks
- The long-run impact of news shocks upon real variables (Y, C, I) is declining over time
- The change in the response of the variables to news shocks is less evident than the variation in the level of uncertainty around the response to news shocks

Summary

- We have provided a means for estimating large $TVP - VAR$ models
- We achieve this by imposing a restriction suggested by the data
- Implementation uses recentering and parameter expansion
- The result is an efficiently computable form